A Closed-Form Slope in the Fed's FIMA-vs-Swap Decision: Cross-Sectional Heterogeneity Through Global-Factor Exposure

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April 24, 2026

Abstract

The Federal Reserve provides dollar liquidity to foreign central banks through two instruments with very different properties: uncollateralized swap lines, granted to a short list of counterparties, and the collateralized FIMA repo facility, open to a larger set. I embed this instrument choice in a three-stage game between the Fed, an alternative liquidity provider (PBOC), and a cross-section of emerging-market counterparties with heterogeneous loadings on a common global-factor stress state. Given a Fed preference weight $\chi \geq 0$ on expected spillover damage in the common-factor stressed state, each country's effective spillover probability becomes a scalar reweighting $P_{\text{eff},i} = \pi_i + \chi\,\omega_i\,\pi_Z$ of its marginal probability π_i by its factor loading ω_i. The main result is a closed-form slope of the optimal Fed eligibility threshold in χ, $\partial\bar{\phi}_i^\chi/\partial\chi = -\Omega_i\,\bar{\phi}_i^\chi\,u_i(\bar{\phi}_i^\chi)/(2A_i^\chi\bar{\phi}_i^\chi + B_i^\chi) < 0$, where $\Omega_i \equiv \omega_i\pi_Z\lambda_i$ scales with each country's loading and all components are observable. The slope is observationally equivalent to a single-country comparative static in π_i; its cross-sectional content is that ω_i varies across counterparties and is separately measurable from Rey-type global-financial-cycle loadings. The ordinal ranking—countries with higher ω_i face lower invoicing thresholds for swap admission—is the paper's robust prediction and holds for every $\chi > 0$. Cardinal magnitudes scale linearly in χ (per unit of χ, the shift is roughly six percentage points for the highest-loading archetype in a five-archetype calibration). A ranking-test specification $\Pr[D_i = 1] = \Phi(\alpha_0 + \alpha_1 M_i + \alpha_2\,\omega_i\phi_{0,i} + \nu_i)$ with $\alpha_1, \alpha_2 > 0$ maps the ordinal prediction to observable Fed eligibility. The partition $\mathcal{E}^* = \{i : \phi_{0,i} > \bar{\phi}_i\}$ under separable spillover follows as a corollary.

Keywords: central bank swap lines, FIMA repo, lender of last resort, dollar invoicing, global financial cycle, reserve-currency competition.

JEL codes: E58, F33, F42, G15.

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1 Introduction

Who should get a Federal Reserve swap line, and who should instead be directed to the FIMA repo facility? The question has been both academically and operationally live since the 2008 and 2020 episodes, when the Fed extended dollar swap lines to a short list of foreign central banks while leaving a much larger group to rely on the collateralized FIMA backstop introduced in 2020 [Choi et al., 2022, Goldberg and Ravazzolo, 2022]. The existing theory of swap-line pricing [Bahaj and Reis, 2022] delivers the Fed's optimal price as a ceiling on covered-interest-parity deviations for a single counterparty, but takes the eligibility set as exogenous. The empirical literature treats eligibility as a reduced-form outcome of political and trade ties [Aizenman et al., 2021]. Neither side of the literature characterizes the Fed's optimal choice between swap and FIMA as a function of counterparty-level observables, and neither addresses the cross-sectional problem the Fed actually faces: a list of countries whose loadings on a common global-factor stress state differ [Rey, 2013, Miranda-Agrippino and Rey, 2020].

This paper provides a closed-form characterization of that cross-sectional problem. I model a three-stage game between the Fed, an alternative liquidity provider (the People's Bank of China), and N emerging-market central banks whose stress shocks are linked to a single common factor through a Bernoulli copula. Given a Fed preference weight $\chi \geq 0$ on expected spillover damage occurring in the common-factor stressed state, each country's *effective spillover probability* in the Fed's expected-loss function becomes a scalar reweighting of its marginal probability,

$$P_{\text{eff},i} \;=\; \pi_i \;+\; \chi\, \omega_i\, \pi_Z,$$

where π_i is the marginal stress probability, ω_i is the country's common-factor loading, and π_Z is the factor's frequency. Under a linear spillover specification with moral-hazard costs on both instruments, the Fed's choice between swap ($E_i = 1$) and FIMA ($E_i = 0$) reduces, country by country, to comparing a country's dollar-invoicing share $\phi_{0,i}$ to a threshold $\bar{\phi}_i^{\chi}$ that is observable-valued. The main result (Theorem 2) is that this threshold has a closed-form slope in χ,

$$\frac{\partial \bar{\phi}_i^{\chi}}{\partial \chi} \;=\; -\,\frac{\Omega_i\, \bar{\phi}_i^{\chi}\, u_i(\bar{\phi}_i^{\chi})}{2A_i^{\chi}\bar{\phi}_i^{\chi} + B_i^{\chi}} \;<\; 0, \qquad \Omega_i \equiv \omega_i \pi_Z \lambda_i, \tag{1}$$

where λ_i is the Fed's spillover sensitivity and $u_i(\bar{\phi}_i^{\chi})$ is the country's uncovered dollar gap at the threshold. The sign is unambiguous: countries with higher common-factor loadings face a lower invoicing bar for swap admission. The ordinal ranking is the paper's robust prediction and holds for every $\chi > 0$. Cardinal magnitudes scale linearly in χ: per unit of χ, a calibrated five-archetype cross-section delivers roughly six percentage points of threshold compression for the highest-ω_i archetype. No cardinal claim is made without the scaling disclosure, because χ is a preference parameter and not externally disciplined.

Mechanism. Given the Fed's preference weight χ on expected spillover in the global-factor state, each country's effective stress probability $P_{\text{eff},i} = \pi_i + \chi\omega_i\pi_Z$ is a scalar function of its loading ω_i. Theorem 2 delivers a closed-form slope in χ that varies cross-sectionally through ω_i. This is observationally equivalent to a single-country comparative static in π_i; the cross-sectional content is the measurement observation that ω_i varies across counterparties and is separately observable from π_i, using existing global-financial-cycle loadings [Rey, 2013, Miranda-Agrippino and Rey, 2020]. The expected-loss aggregator does *not* penalize simultaneous stress separately—cross-country coincidence does not enter Fed welfare as a distinct term. Instead, χ is the weight the Fed places on each country's marginal conditional-excess probability under the common factor. The channel flows exclusively through the Fed's spillover term; the PBOC's best response $S_i^*(0)$ does not depend on χ, which is why the slope factors cleanly into $\bar{\phi}_i^\chi \cdot u_i(\bar{\phi}_i^\chi)$.

The role of χ. The weight χ is a normative parameter of the Fed's loss function, not a positive prediction about how the Fed aggregates stress in practice. It captures the extra weight placed on the marginal conditional-excess stress probability under the common factor, not a correlation channel. Because χ has no external empirical anchor, cardinal magnitudes are reported as slopes per unit of χ rather than at any single benchmark. The ordinal cross-sectional prediction—higher-ω_i countries face lower thresholds—holds for every $\chi > 0$ and is the paper's robust content.

Literature. The paper sits at the intersection of several strands. Most directly, it extends the single-country pricing theory of Bahaj and Reis [2022] to a cross-section and adds a closed-form comparative static in the Fed's common-factor preference χ. The welfare framework builds on Kekre and Lenel [2024] and Kekre and Lenel [2025], who analyze dollar swap provision in general equilibrium with a single provider. On the international lender-of-last-resort side, Bocola and Lorenzoni [2020] models dollarization with a single LOLR in a closed economy with multiple equilibria; I generalize to a two-provider setting across a heterogeneous cross-section. The strategic Fed-PBOC interaction has a natural reading in the Farhi and Maggiori [2018] framework of reserve-currency competition; I include PBOC as an exogenous competing backstop, and characterize in Appendix D the regime in which the strategic channel is first-order (the calibration places the paper outside that regime).

On the invoicing side, Gopinath et al. [2020] and Gopinath and Stein [2021] document the role of dollar invoicing in trade and finance, and Chahrour and Valchev [2022] and Mukhin [2022] formalize the complementarities generating multiple invoicing equilibria. Bahaj and Reis [2026] causally identifies PBOC swap lines as shifting RMB invoicing, which disciplines the invoicing-drift parameter γ_i. On the PBOC side, Horn et al. [2023] and Clayton et al. [2025] document the scale and objective of China's bilateral lending. On the global-factor side, Rey [2013] and Miranda-Agrippino and Rey [2020] provide the measurement of ω_i that makes (1) empirically implementable.

Methodologically, the paper's welfare criterion draws on the mechanism-design LOLR tradition of Rochet and Vives [2004] and Bengui and Bianchi [2019], who characterize optimal coverage in single-provider settings with commitment, and Farhi and Tirole [2025], who studies domestic support policies with global firms. The weight χ is a reduced-form device for the Fed's preferential weight on conditional-excess stress in the common-factor state; microfoundations rooted in CVaR or spectral risk aggregators would deliver a true coincidence-penalizing aggregator (and would plausibly deserve the "super-additivity" label in a stronger sense), at the cost of losing the paper's closed-form slope.

Scope and honest framing. The paper delivers a characterization, not a full positive theory of Fed swap-line policy. Several limitations are explicit throughout. First, χ is a normative preference parameter with no external anchor; cardinal magnitudes are reported per unit of χ, and the ordinal cross-sectional prediction is the robust content. Second, the expected-loss aggregator carries cross-country covariance only as a scalar per-country reweighting, not as genuine coincidence-penalizing aggregation. Third, several primitives (κ, β, μ, ξ_i) are not point-identified; I therefore propose an ordinal-consistency specification rather than a structural logit on cardinal thresholds. Fourth, the baseline assumes separable PBOC capacity across counterparties; the shared-capacity extension is sketched but not solved. Fifth, PBOC enters as an exogenous competing backstop; at the baseline calibration, its strategic channel is second-order. Section 4.6 treats each limitation directly.

Roadmap. Section 2 sets up the game, the one-factor stress structure, and the binding-regime conditions. Section 3 states and proves the baseline single-country threshold (Theorem 1), the main common-factor slope (Theorem 2), and the partition corollary (Proposition 1). Section 4 reports the archetype calibration, sensitivity to unidentified weights, the ranking-test specification, testable predictions, and policy implications. Section 5 concludes. The appendix contains all proofs and, in a consolidated extensions section, the Cournot regime diagnostic and the shared-capacity sketch.

2 Model

2.1 Agents, timing, and primitives

There are $N+2$ strategic players: the Federal Reserve, the PBOC, and a set $\mathcal{I} = \{1, \ldots, N\}$ of emerging-market central banks. Each country i is described by an observable primitive vector

$$\theta_i \equiv (\phi_{0,i},\, d_{0,i},\, h_i,\, \pi_i,\, \lambda_i,\, \gamma_i,\, \xi_i,\, \theta_i^{\text{wedge}}),$$

while the PBOC-side primitives $(\rho, \mu, \sigma, \bar{S})$ are common. Here $\phi_{0,i}$ is country i's initial dollar-invoicing share, $d_{0,i}$ its bilateral dollar trade exposure, h_i its FIMA haircut, π_i the marginal probability of an idiosyncratic dollar-funding stress episode, λ_i the Fed spillover sensitivity, γ_i an invoicing-drift parameter, ξ_i a Cournot-interaction coefficient, and θ_i^{wedge} the firm-level RMB substitutability of the FIMA haircut residual.

The game has three stages.

Stage 1 (ex-ante commitment, $t = 0$). Fed and PBOC move simultaneously. The Fed chooses, for each country, (E_i, p_i, h_i): eligibility $E_i \in \{0, 1\}$, swap price $p_i \geq 0$, and FIMA haircut $h_i \geq 0$. The central policy object is the eligibility set $\mathcal{E}^* \subset \mathcal{I}$ of countries with $E_i = 1$; the complement receives FIMA only. The PBOC chooses country-level RMB swap capacities $\{S_i\}_{i \in \mathcal{I}}$, with $S_i \in [0, \bar{S}]$, and RMB prices $q_i \geq 0$. The PBOC pays country-separable cost $\sigma S_i^2/2$; the shared-capacity extension is deferred to Appendix D.2.

Stage 2 (stress, $t = 1$). A common-factor shock $Z \in \{0, 1\}$ realizes with $\Pr(Z = 1) = \pi_Z \in [0, 1)$. Conditional on Z, country i's stress indicator $\epsilon_i \in \{0, 1\}$ is drawn so that the marginal is $\Pr(\epsilon_i = 1) = \pi_i$ and the loading ω_i controls the covariance structure:

$$\mathrm{Cov}(\epsilon_i, \epsilon_j) \;=\; \omega_i \omega_j \pi_Z (1 - \pi_Z), \qquad \omega_i \;\equiv\; \frac{\Pr(\epsilon_i = 1 \mid Z = 1) - \pi_i}{1 - \pi_Z}. \tag{2}$$

This is the standard single-factor Bernoulli copula. Ensuring both conditional probabilities lie in $[0, 1]$ gives the admissible support

$$\omega_i \in \big[\, 0,\ \min\!\big\{\pi_i/\pi_Z,\ (1 - \pi_i)/(1 - \pi_Z)\big\}\,\big]\,. \tag{3}$$

At $\omega_i = 0$, country-i stress is independent of the factor. Country i's dollar funding gap is $D_i(\phi_{0,i}) = d_{0,i}\phi_{0,i}$.

Stage 3 (invoicing update, $t = 2$). Country-i stress (probability π_i) shifts country-i invoicing according to

$$\phi_{1,i} - \phi_{0,i} \;=\; -\gamma_{\mathrm{raw},i} \cdot [x_{P,i}^{\mathrm{stress}} - x_{F,i}^{\mathrm{stress}}] \quad \text{(under country-}i\text{ stress)},$$

with $\gamma_{\mathrm{raw},i} > 0$. Taking unconditional expectation absorbs π_i:

$$\mathbb{E}[\phi_{1,i} - \phi_{0,i}] \;=\; -\gamma_i \cdot [x_{P,i}^{\mathrm{stress}} - x_{F,i}^{\mathrm{stress}}], \qquad \gamma_i \equiv \pi_i \gamma_{\mathrm{raw},i}. \tag{4}$$

Write $\dot{\phi}_i(E_i, S_i) \equiv \mathbb{E}[\phi_{1,i} - \phi_{0,i}]$.

2.2 Spillovers, moral hazard, and stress draws

Country-i stress generates a US financial-stability cost $\Lambda_i(\phi_{0,i}, u_i) = \lambda_i \phi_{0,i} \psi(u_i)$, where u_i is country i's uncovered dollar gap, $\lambda_i > 0$, and ψ is strictly convex with $\psi(0) = 0$, $\psi' > 0$.

Assumption 1 (H1)**.** ψ is convex ($\psi'' \geq 0$) and strictly increasing ($\psi' > 0$) on $\mathbb{R}_+$, with $\psi(0) = 0$.

The linear case $\psi(u) = u$ is the boundary case of H1, under which Theorems 1 and 2 admit closed-form expressions. For strictly convex ψ, the characterization is qualitatively preserved (monotone comparative statics; existence and uniqueness of a threshold); closed-form roots are not available in that case. Appendix C states the strictly-convex extension.

Moral hazard on the swap instrument is $\mu_{\text{sw},i}(p_i) > 0$; on FIMA it is $\eta_i(h_i) = \eta_{0,i}(1-h_i)/(1+h_i)$, with $\eta_{0,i} \in (0, \mu_{\text{sw},i}(p_i))$ so FIMA carries less moral hazard than swap at any $h_i > 0$. The total moral-hazard cost is

$$MH_i(E_i, p_i, h_i) \;=\; E_i\mu_{\text{sw},i}(p_i) + (1 - E_i)\eta_i(h_i). \tag{5}$$

The Fed's invoicing-preservation weight is $\beta \geq 0$, country-invariant. A fraction θ_i^{wedge} of the FIMA haircut residual is RMB-substitutable at the firm level; write θ_i when context is unambiguous.

Given $p_i \leq q_i < m_i$, country-i stress-state draws are

$$x^{\text{stress}}_{F,i} = E_i d_{0,i}\phi_{0,i} + (1 - E_i)\frac{d_{0,i}\phi_{0,i}}{1 + h_i}, \qquad x^{\text{stress}}_{P,i} = \min\{S_i,\ \theta_i(1 - E_i)K_i\}, \tag{6}$$

with $K_i \equiv d_{0,i}\phi_{0,i}h_i/(1 + h_i)$.

Assumption 2 (Binding)**.** $S_i \leq \theta_i(1 - E_i)K_i$ at the equilibrium choice of S_i for each country on the non-swap side.

Under Assumption 2, $x^{\text{stress}}_{P,i} = S_i$ and the uncovered gap is

$$u_i(E_i, S_i, \phi_{0,i}) \;=\; (1 - E_i)K_i - S_i. \tag{7}$$

At the $E_i = 1$ corner, $x^{\text{stress}}_{P,i} = 0$ and $u_i = 0$. Section 4 verifies the binding assumption numerically across all calibrated archetypes, with a cushion factor exceeding four.

2.3 Objectives

The PBOC's country-i objective is

$$W_i^P(S_i; E_i, \phi_{0,i}) \;=\; \rho\pi_i(1 - \xi_i E_i)\cdot x^{\text{stress}}_{P,i} \;-\; \mu\dot{\phi}_i(E_i, S_i) \;-\; \tfrac{\sigma}{2}S_i^2. \tag{8}$$

The first term is PBOC's private benefit from RMB usage in stress, scaled down by ξ_i when the Fed provides a swap (partial substitution). The second is the PBOC's valuation of invoicing shift toward RMB. For simplicity, I assume PBOC's objective is expected-loss-linear in stress probability and carries no super-additivity weight; this asymmetry with the Fed's objective reflects differing institutional mandates (PBOC focused on the direct invoicing response, the Fed internalizing co-incident crisis losses through χ). The independence of the slope (16) from χ on the PBOC side follows from this modeling choice, not from a separate economic argument. Maximizing (8) in the binding regime with $E_i < 1$ yields

$$S_i^*(E_i) \;=\; \frac{\rho\pi_i(1 - \xi_i E_i) + \mu\gamma_i}{\sigma}, \qquad S_i^*(0) = \frac{\rho\pi_i + \mu\gamma_i}{\sigma}. \tag{9}$$

At $E_i = 1$, $x^{\text{stress}}_{P,i} = 0$ and the corner $S_i^*(1)$ drops out of Fed welfare.

The Fed's aggregate welfare separates under A-Sep (see Section 3) into a sum over countries:

$$W_i^F(E_i, S_i; \phi_{0,i}) \;=\; -\pi_i \lambda_i \phi_{0,i} \psi(u_i(E_i, S_i, \phi_{0,i})) \;-\; \kappa MH_i(E_i, p_i, h_i) \;+\; \beta \dot{\phi}_i(E_i, S_i), \tag{10}$$

where $\kappa \geq 0$ is the Fed's moral-hazard weight. Total Fed welfare is $W^F(\mathcal{E}) = \sum_i W_i^F(E_i, S_i^*(E_i); \phi_{0,i})$.

2.4 Information and equilibrium

All primitives $\{\theta_i\}$ and $(\rho, \mu, \sigma, \bar{S}, \pi_Z, \chi)$ are common knowledge. The solution concept is subgame-perfect Nash equilibrium. For each country, the Fed picks $E_i \in \{0, 1\}$ to maximize $W_i^F(E_i, S_i^*(E_i); \phi_{0,i})$, taking PBOC's best response (9) as given.

2.5 Separability and the one-factor aggregator

Two Fed spillover aggregators are in play. The baseline, used for Theorem 1 and Proposition 1, is separable:

Assumption 3 (A-Sep)**.** Fed aggregate spillover cost is a sum of country-level terms, $\Lambda(\mathcal{E}) = \sum_{i \in \mathcal{I}} \pi_i \lambda_i \phi_{0,i} \psi(u_i(E_i, S_i^*(E_i), \phi_{0,i}))$.

Theorem 2 replaces A-Sep with a Fed welfare criterion that applies a super-additivity weight $\chi \geq 0$ to the covariance contribution to expected spillover. Formally, define the baseline (A-Sep) spillover as $\Lambda^0(\mathcal{E}) = \sum_i \lambda_i \phi_{0,i} \psi(u_i) \mathbf{1}\{\epsilon_i = 1\}$ and the super-additivity correction as the extra cost the Fed assigns to stress occurring in the common-factor state,

$$\Delta\Lambda(\mathcal{E}, Z) \;\equiv\; \mathbf{1}\{Z = 1\} \sum_i \omega_i \, \lambda_i \, \phi_{0,i} \, \psi(u_i) \, \mathbf{1}\{\epsilon_i = 1\}.$$

The Fed's effective spillover objective is

$$\mathbb{E}_\chi[\Lambda] \;\equiv\; \mathbb{E}[\Lambda^0(\mathcal{E})] \;+\; \chi \, \mathbb{E}[\Delta\Lambda(\mathcal{E}, Z)]. \tag{11}$$

Here $\chi = 0$ recovers A-Sep (the Fed weights stress states additively regardless of the common factor). For $\chi > 0$, the Fed places additional weight on the covariance contribution, that is, on the extent to which country-i stress co-occurs with the global-factor state $Z = 1$. Computing the two expectations, $\mathbb{E}[\Lambda^0] = \sum_i \pi_i \lambda_i \phi_{0,i} \psi(u_i)$ (using $\Pr(\epsilon_i = 1) = \pi_i$), and $\mathbb{E}[\Delta\Lambda] = \sum_i \omega_i \pi_Z \lambda_i \phi_{0,i} \psi(u_i)$ (using the copula definition of ω_i in (2); see Appendix B for the derivation). Collecting terms,

$$\mathbb{E}_\chi[\Lambda] \;=\; \sum_i P_{\text{eff},i} \, \lambda_i \, \phi_{0,i} \, \psi(u_i), \qquad P_{\text{eff},i} \;\equiv\; \pi_i + \chi \, \omega_i \, \pi_Z. \tag{12}$$

Under (12), Fed expected welfare separates into a sum over countries,

$$W^F(\mathcal{E}) \;=\; \sum_i \Big[-P_{\text{eff},i} \lambda_i \phi_{0,i} \psi(u_i) \;-\; \kappa MH_i \;+\; \beta \dot{\phi}_i \Big].$$

$P_{\text{eff},i}$ is country i's *effective spillover probability* under the Fed's super-additivity criterion: the baseline marginal π_i plus a common-factor term $\chi\omega_i\pi_Z$ that is nonzero only for countries with positive global-factor loading. The χ weight is a Fed preference parameter; its role is to convert the covariance structure of $\{\epsilon_i, Z\}$ into a scalar reweighting of country-i expected spillover.

3 Results

The paper's main result (Theorem 2) is the closed-form slope of the Fed's country-i eligibility threshold in the Fed's super-additivity weight χ. Section 3.1 states the single-country threshold (Theorem 1) as a baseline: it reproduces the Bahaj and Reis [2022] pricing tradeoff in a form the cross-sectional analysis will nest. Section 3.2 states Theorem 2. Section 3.4 states the partition corollary. Section 3.5 reports the cross-sectional comparative statics.

3.1 Single-country threshold

Fix p with $\mu_{\text{sw}}(p) > \eta(h)$. Drop the i subscript. Under $\psi(u) = u$ and Assumption 2, define

$$A \;=\; \pi\lambda\frac{d_0 h}{1+h}, \qquad B \;=\; \beta\gamma d_0\frac{h}{1+h} - \pi\lambda S^*(0), \qquad \tilde{C} \;=\; \kappa[\mu_{\text{sw}}(p) - \eta(h)] - \beta\gamma S^*(0).$$

Theorem 1 (Single-country threshold)**.** *Under H1, Assumption 2, $\psi(u) = u$, and PBOC best response $S^*(0) = [\rho\pi + \mu\gamma]/\sigma$ at $E = 0$, the Fed strictly prefers $E = 1$ to $E = 0$ if and only if $\phi_0 > \bar{\phi}$, where*

$$\bar{\phi} \;=\; \frac{-B + \sqrt{B^2 + 4A\tilde{C}}}{2A}. \tag{13}$$

The proof is in Appendix A. The coefficient $A > 0$ scales spillover in ϕ_0^2; $\tilde{C}$ is the swap-side moral-hazard wedge net of a PBOC-capacity rebate $\beta\gamma S^*(0)$; B collects the linear-in-ϕ_0 terms, including a subtractive $\pi\lambda S^*(0)$ contribution from PBOC's liquidity coverage that shrinks the Fed's spillover at $E = 0$. The positive root exists uniquely under $A > 0$ and $\tilde{C} > 0$. Countries with $\tilde{C} \le 0$ (the swap-side moral-hazard wedge is dominated by the PBOC rebate) are always assigned FIMA; they are excluded from the threshold characterization and from the partition in Proposition 1.

Remark 1. Theorem 1 recovers the Bahaj and Reis [2022] pricing tradeoff for a single country with a *choice* between FIMA and swap. In that paper, eligibility is taken as given and the problem is the swap price; here, eligibility is the object of choice and the threshold is expressed in the country's invoicing share.

3.2 The common-factor slope

Replace A-Sep with the super-additivity criterion (11). Under the expected-value representation (12), repeat the derivation of Theorem 1 with π_i replaced by $P_{\text{eff},i}$ in the spillover term only. The threshold coefficients become

$$A_i^\chi = P_{\text{eff},i}\lambda_i \frac{d_{0,i}h_i}{1+h_i}, \qquad B_i^\chi = \beta\gamma_i d_{0,i}\frac{h_i}{1+h_i} - P_{\text{eff},i}\lambda_i S_i^*(0), \qquad \tilde{C}_i^\chi = \kappa[\mu_{\text{sw},i}(p_i) - \eta_i(h_i)] - \beta\gamma_i S_i^*(0). \tag{14}$$

Theorem 2 (Common-factor slope)**.** *Under the super-additivity criterion* (11)*, $\psi(u) = u$, H1, Assumption 2 at $E_i = 0$, and the hypotheses $A_i^\chi > 0$ and $\tilde{C}_i^\chi > 0$ (guaranteeing a positive root $\bar{\phi}_i^\chi \in (0,\infty)$), country i's eligibility threshold $\bar{\phi}_i^\chi$ is the positive root of*

$$A_i^\chi(\bar{\phi}_i^\chi)^2 + B_i^\chi \bar{\phi}_i^\chi - \tilde{C}_i^\chi \;=\; 0, \tag{15}$$

with coefficients (14)*. For every country i with $\omega_i > 0$,*

$$\frac{\partial \bar{\phi}_i^\chi}{\partial \chi} \;=\; -\frac{\Omega_i \cdot \bar{\phi}_i^\chi \cdot u_i(\bar{\phi}_i^\chi)}{2A_i^\chi \bar{\phi}_i^\chi + B_i^\chi} \;<\; 0, \qquad \Omega_i \equiv \omega_i \pi_Z \lambda_i, \tag{16}$$

where $u_i(\bar{\phi}_i^\chi) = d_{0,i}h_i\bar{\phi}_i^\chi/(1+h_i) - S_i^*(0) > 0$ is country i's uncovered dollar gap evaluated at the threshold.

The proof is in Appendix B. The content of Theorem 2 is not the existence of a country-by-country threshold. Under the super-additivity criterion, Fed expected welfare separates into a sum over countries, so the threshold characterization follows from Theorem 1 applied country by country with π_i replaced by $P_{\text{eff},i}$. The content is the closed-form slope in χ: a cross-sectional prediction that moves with each country's observable factor loading ω_i.

Mechanism. Under the Fed's expected-loss criterion with preference weight χ, a country's common-factor loading ω_i raises its effective stress probability via $P_{\text{eff},i} = \pi_i + \chi\omega_i\pi_Z$, a scalar reweighting. This produces a cross-sectional slope: countries with higher ω_i face a lower FIMA-vs-swap threshold because their expected spillover, weighted by χ, is larger. The derivation of the slope in χ follows from an algebraic identity on the quadratic root (implicit differentiation of (15)); the underlying economic force is the Fed's preferential weight on conditional-excess stress, which is a normative parameter of the loss function, not a positive prediction about Fed behavior or a covariance-driven interaction. The slope factor $\bar{\phi}_i^\chi \cdot u_i(\bar{\phi}_i^\chi)$ is the product of invoicing at the threshold and the uncovered dollar gap at $E_i = 0$, each directly observable. The amplification operates exclusively through the Fed's spillover term, not through PBOC's best response $S_i^*(0)$, because the PBOC's objective excludes the super-additivity term by assumption (Section 2).

A single-country model with reweighted probability $\pi \to \pi + \chi\omega\pi_Z$ produces the same slope formula; the cross-sectional content of Theorem 2 is that ω_i varies across countries and is separately observable from π_i using Rey-type global-financial-cycle loadings, and that the same Fed agent applies χ uniformly across its counterparties.

Partition boundary and who switches. At the threshold, $\partial\bar{\phi}_i^\chi/\partial\chi < 0$ scales with $\Omega_i = \omega_i\pi_Z\lambda_i$ and with the country's uncovered gap $u_i(\bar{\phi}_i^\chi)$. Countries at or near the partition boundary ($\phi_{0,i} \approx \bar{\phi}_i^\chi$) are the most sensitive to changes in the Fed's χ: as χ rises, high-ω_i borderline countries cross from FIMA to swap first. Among the five calibrated archetypes (Section 4.1), archetype A5 (low ω, comfortably in FIMA) remains stable under χ-variation; archetype A4 (moderate ω, marginal in the partition) is the most sensitive, and cardinally large enough χ moves A4 further into swap while A1 (highest ω) receives the largest threshold shift per unit of χ.

Remark 2 (Coefficient allocation)**.** $P_{\text{eff},i}$ enters A_i^χ and the $P_{\text{eff},i}\lambda_i S_i^*(0)$ piece of B_i^χ, but *not* $\tilde{C}_i^\chi$. Moral-hazard and invoicing-preservation constants do not multiply spillover probability, so the χ-derivative of $\tilde{C}_i^\chi$ is zero. Misallocating $P_{\text{eff},i}$ into $\tilde{C}_i^\chi$ would introduce a spurious correction; under the correct allocation, the comparative static collapses to (16).

3.3 Taylor expansion and accuracy

Expanding $\bar{\phi}_i^\chi$ around $\chi = 0$,

$$\bar{\phi}_i^\chi - \bar{\phi}_i \;=\; -\frac{\omega_i\pi_Z\lambda_i}{2A_i\bar{\phi}_i + B_i}\cdot\bar{\phi}_i\cdot u_i(\bar{\phi}_i)\cdot\chi \;+\; O(\chi^2). \tag{17}$$

The first-order magnitude scales linearly in χ, π_Z, ω_i, λ_i, and $\bar{\phi}_i \cdot u_i(\bar{\phi}_i)$. At $\chi = 1$ and the high-ω archetype of Section 4.1 (A1 with $\omega_1 = 0.80$), the first-order Taylor prediction is -8.7 percentage points, whereas the exact positive root of (15) at $P_{\text{eff},1} = 0.12$ yields $\bar{\phi}_1^\chi = 0.324$, i.e., a shift of -6.4 percentage points. The first-order linearization overstates the exact shift by roughly 35% in this region because the root is concave in χ.[1]

3.4 Partition consequence

Proposition 1 (Cross-sectional partition)**.** *Under A-Sep (equivalently $\chi = 0$), H1, Assumption 2 at $E_i = 0$ for every i, $\psi(u) = u$, PBOC country-separability, and $\tilde{C}_i > 0$ for all $i \in \mathcal{I}$, the Fed's optimal eligibility set is*

$$\mathcal{E}^* \;=\; \{\, i \in \mathcal{I} \,:\, \phi_{0,i} > \bar{\phi}_i \,\}, \tag{18}$$

where $\bar{\phi}_i$ is the single-country threshold (13) evaluated at country-i primitives.

Proof. Under A-Sep, $W^F(\mathcal{E}) = \sum_i W_i^F(E_i, S_i^*(E_i); \phi_{0,i})$, so $\max_{\mathcal{E}} W^F = \sum_i \max_{E_i\in\{0,1\}} W_i^F(E_i, S_i^*(E_i); \phi_{0,i})$. Each country-level binary choice applies Theorem 1 verbatim to country-i primitives: $E_i = 1$ if and only if $\phi_{0,i} > \bar{\phi}_i$. Combining yields (18). □

Proposition 1 is a direct corollary of A-Sep, Theorem 1 applied N times, and summation. Countries with $\tilde{C}_i \le 0$ (dominated by the PBOC rebate) do not have a well-defined threshold root and are always assigned FIMA at $\phi_{0,i} > 0$; they are excluded from (18). The substantive content of the partition is A-Sep itself; the theorem-level content of the paper is the χ-slope of Theorem 2.

[1] Throughout the calibration in Section 4.1 I report the exact root rather than the first-order expansion.

3.5 Cross-sectional comparative statics

Implicit differentiation of (15) at general $\chi \geq 0$ yields the signed comparative statics in Table 1, verified symbolically at the A1 baseline of Section 4.1 and numerically across all five archetypes. The ω_i and χ signs are the new content (Theorem 2); the remaining signs echo the single-country comparative statics under A-Sep.

Parameter ζ	$\text{sgn}(\partial\bar{\phi}_i^\chi/\partial\zeta)$	Mechanism
ω_i	$-$	Common-factor loading raises effective spillover probability (Theorem 2).
χ	$-$ (if $\omega_i > 0$)	Fed super-additivity weight on common-factor stress (Theorem 2).
π_Z	$-$ (if $\omega_i > 0$)	Factor frequency raises $P_{\text{eff},i}$.
$\phi_{0,j}$, $j \neq i$	0	Country-level decoupling under the super-additivity criterion.
π_i	$-$	Raises A_i^χ and the PBOC rebate in B_i^χ.
λ_i	$-$	Higher spillover sensitivity raises A_i^χ.
$d_{0,i}$	$-$	Higher trade exposure scales both A_i^χ and invoicing drift.
h_i	$-$	Haircut enlarges spillover scaling and invoicing-drift cost.
γ_i	$-$	Invoicing sensitivity raises drift term and PBOC rebate.
κ	$+$	Higher moral-hazard weight enlarges FIMA's advantage.
β	$-$ (weak)	Invoicing preservation tilts toward swap.
ρ	$+$ (via $S_i^*(0)$)	PBOC private marginal benefit raises the rebate.
σ	$-$	Lowers $S_i^*(0)$ and the rebate.

Table 1: Cross-sectional comparative statics of the country-i eligibility threshold.

Figure 1 plots the leading four comparative statics numerically at A1 baseline primitives.

4 Calibration, Identification, and Discussion

This section implements the threshold-slope characterization at a calibrated cross-section of five archetypes (Section 4.1), examines sensitivity to unidentified weights (Section 4.2), develops the ranking-test empirical specification (Section 4.3), collects testable predictions (Section 4.4), discusses policy (Section 4.5), and states limitations (Section 4.6).

4.1 Archetype calibration

I fix country-invariant primitives at $\lambda = 1$, $\kappa = 0.05$, $\mu_{\text{sw}} = 0.01$, $\eta_0 = 0.003$, $\beta = 0.2$, $\gamma = 0.02$, $\rho = 0.001$, $\sigma = 10$, $\mu = 0.5$, $\xi = 0.3$, $\pi_Z = 0.05$. The country-specific primitives $(\phi_{0,i}, d_{0,i}, h_i, \pi_i, \omega_i)$ define five archetypes spanning the empirically plausible parameter space. Archetype labels are illustrative parameter descriptors; they are not estimated from named countries.

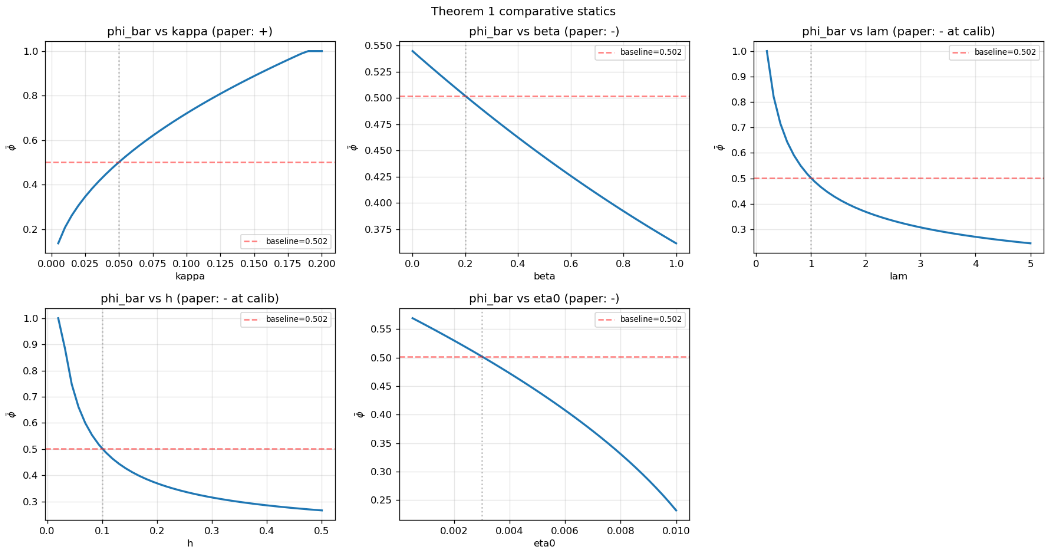

Figure 1: Theorem 1/2 comparative statics at A1 baseline. Each panel varies one primitive while holding the remainder fixed. Signs match Table 1.

- **A1 (high invoicing, high common factor):** $\phi_{0,i} = 0.80$, $d_{0,i} = 0.40$, $h_i = 0.08$, $\pi_i = 0.08$, $\omega_i = 0.80$.
- **A2 (moderate invoicing, moderate common factor):** $\phi_{0,i} = 0.65$, $d_{0,i} = 0.25$, $h_i = 0.12$, $\pi_i = 0.07$, $\omega_i = 0.40$.
- **A3 (low invoicing, high idiosyncratic stress, low common factor):** $\phi_{0,i} = 0.55$, $d_{0,i} = 0.25$, $h_i = 0.20$, $\pi_i = 0.25$, $\omega_i = 0.10$.
- **A4 (low invoicing, low stress, moderate common factor):** $\phi_{0,i} = 0.55$, $d_{0,i} = 0.20$, $h_i = 0.15$, $\pi_i = 0.05$, $\omega_i = 0.30$.
- **A5 (low invoicing, low common factor):** $\phi_{0,i} = 0.35$, $d_{0,i} = 0.18$, $h_i = 0.18$, $\pi_i = 0.06$, $\omega_i = 0.25$.

All five archetype loadings ω_i satisfy the admissibility constraint (3). Table 2 summarizes the threshold computation. Figure 2 visualizes the resulting partition.

Archetype	$\phi_{0,i}$	$d_{0,i}$	h_i	π_i	ω_i	$S_i^*(0)$	A_i	B_i	$\tilde{C}_i$	$\bar{\phi}_i$	Margin	As
A3	0.55	0.25	0.20	0.25	0.10	1.03×10^{-3}	1.04×10^{-2}	-8.96×10^{-5}	4.00×10^{-4}	0.200	+0.350	
A5	0.35	0.18	0.18	0.06	0.25	1.01×10^{-3}	1.65×10^{-3}	$+4.95\times10^{-5}$	3.92×10^{-4}	0.473	−0.123	
A4	0.55	0.20	0.15	0.05	0.30	1.01×10^{-3}	1.30×10^{-3}	$+5.41\times10^{-5}$	3.89×10^{-4}	0.526	+0.024	
A2	0.65	0.25	0.12	0.07	0.40	1.01×10^{-3}	1.88×10^{-3}	$+3.67\times10^{-5}$	3.82×10^{-4}	0.442	+0.208	
A1	0.80	0.40	0.08	0.08	0.80	1.01×10^{-3}	2.37×10^{-3}	$+3.79\times10^{-5}$	3.72×10^{-4}	0.388	+0.412	

Table 2: Five-archetype threshold calibration under A-Sep ($\chi = 0$). Sorted by ω_i. Margin $= \phi_{0,i} - \bar{\phi}_i$; positive margin assigns the country to swap under Proposition 1.

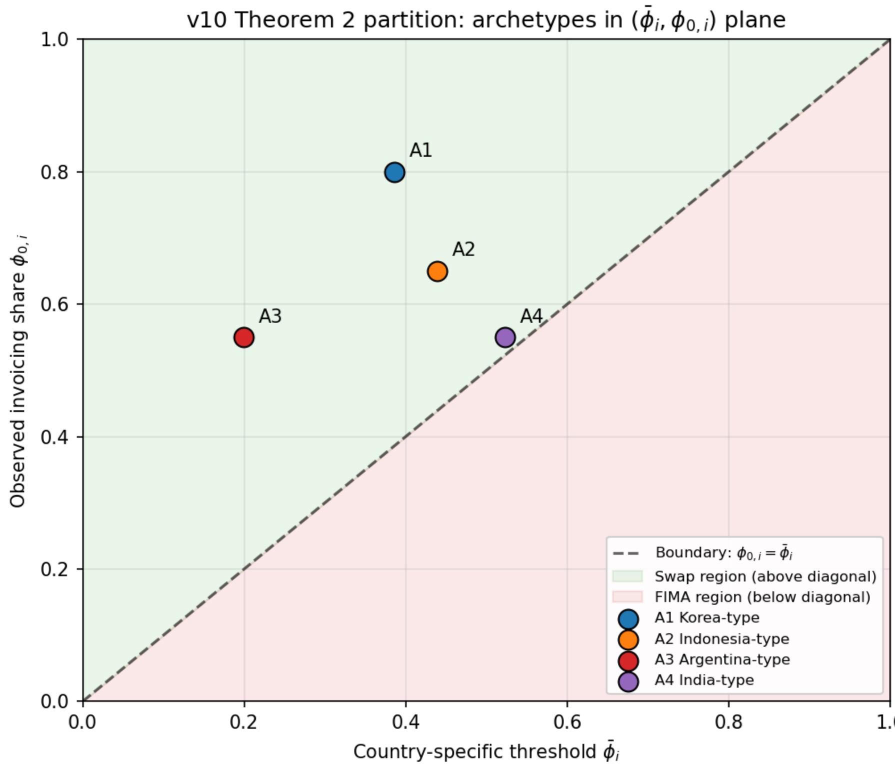

Figure 2: Partition of the five archetypes. Each point is $(\bar{\phi}_i, \phi_{0,i})$; the 45-degree line separates swap (above) from FIMA (below).

The partition (18) is non-trivial: A1–A4 are assigned to swap and A5 to FIMA. Archetype A3, despite low invoicing $\phi_{0,3} = 0.55$, clears its threshold comfortably because a high idiosyncratic stress probability $\pi_3 = 0.25$ pushes A_3 up. A4 is marginal, with a +2.4 percentage-point cushion. A5's threshold ($\bar{\phi}_5 = 0.473$) exceeds its invoicing ($\phi_{0,5} = 0.35$), placing it on the FIMA side.

The binding condition of Assumption 2 is satisfied with a cushion of at least 4 across all archetypes: $\theta_i K_i / S_i^*(0) \in [4, 10]$ at all five points. The uncovered gap $u_i(\bar{\phi}_i) > 0$ in Theorem 2 is positive at every archetype.

Exact common-factor correction at $\chi = 1$

At the illustrative $\chi = 1$ benchmark, the effective probability $P_{\text{eff},i} = \pi_i + \omega_i \pi_Z$ rises by $\omega_i \pi_Z$, and the exact threshold is computed from the positive root of (15). The $\chi = 1$ benchmark has no structural interpretation; cardinal magnitudes scale linearly in χ, so I report the result as a per-unit-of-χ shift. For A1 at $\omega_1 = 0.80$, $P_{\text{eff},1} = 0.12$:

$$A_1^\chi = 0.0035556, \quad B_1^\chi = -2.44\times10^{-6}, \quad \tilde{C}_1^\chi = 3.72\times10^{-4}, \quad \bar{\phi}_1^\chi = 0.324.$$

The exact shift is -6.4 percentage points per unit of χ. For A3 at $\omega_3 = 0.10$, $P_{\text{eff},3} = 0.255$:

$$A_3^\chi = 0.010625, \quad B_3^\chi = -9.47\times10^{-5}, \quad \tilde{C}_3^\chi = 4.00\times10^{-4}, \quad \bar{\phi}_3^\chi = 0.1984.$$

The exact shift is approximately -0.2 percentage points per unit of χ. The cross-sectional difference-in-ω shift between the highest- and lowest-ω archetype is roughly six percentage points per unit of χ, which is the magnitude Theorem 2 predicts. Figure 3 plots $\bar{\phi}_i^\chi$ against ω_i across archetypes.

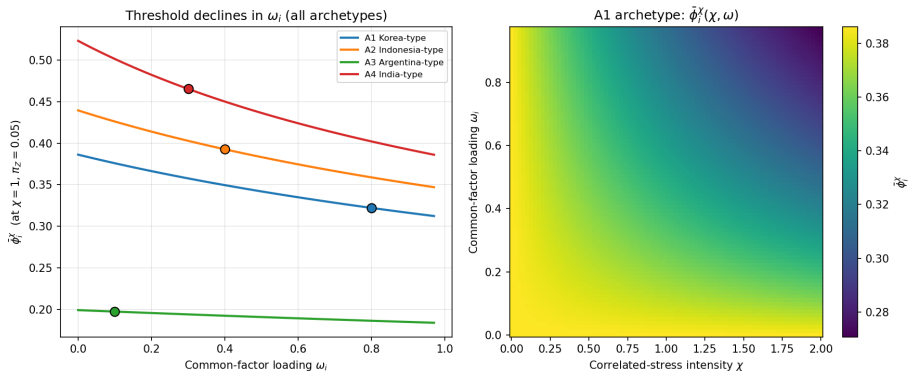

Figure 3: Country-i eligibility threshold $\bar{\phi}_i^\chi$ as a function of common-factor loading ω_i at $\chi = 1$, across archetypes and neighboring perturbations. Higher ω_i compresses the threshold.

Transition analysis

Suppose every country's invoicing share declines by 5 percentage points (a stylized dollar-invoicing erosion). A country transitions from swap to FIMA if and only if its margin $\phi_{0,i} - \bar{\phi}_i < 0.05$. From Table 2: A1 (margin 0.412), A2 (margin 0.208), and A3 (margin 0.350) remain in swap. A4 (margin 0.024) transitions to FIMA. A5 is already in FIMA. The at-risk group is the low-π_i, low-$d_{0,i}$ profile with moderate initial invoicing; the high-π_i profile (A3) is anchored in swap by the spillover term.

4.2 Sensitivity to unidentified weights

Several primitives in the Fed and PBOC objectives (κ, β, μ, ξ_i) are not point-identified from revealed Fed eligibility. I therefore perturb each by $\pm 50\%$ and track the resulting thresholds for the most sensitive archetypes. Table 3 reports the results.

Archetype	$\bar{\phi}_i$ base	$\kappa = 0.025$	$\kappa = 0.075$	$\beta = 0.10$	$\beta = 0.30$	$\mu = 0.25$	$\mu = 0.75$	Assignment change
A1	0.388	0.269	0.475	0.400	0.373	0.388	0.388	None
A4	0.526	0.371	0.645	0.542	0.510	0.526	0.526	Flips to FIMA at κ=0.075
A5	0.473	0.338	0.589	0.488	0.459	0.473	0.473	Flips to swap at κ=0.025

Table 3: Sensitivity of $\bar{\phi}_i$ to $\pm 50\%$ perturbations of the unidentified weights. Baseline parameters as in Section 4.1.

The moral-hazard weight κ is the dominant sensitivity: a $+50\%$ increase shifts thresholds by 9 to 12 percentage points and can flip the marginal archetype A4 from swap to FIMA; a -50% decrease can flip A5 from FIMA to swap. The invoicing-preservation weight β has modest effects (1 to 2 percentage points). The PBOC primitives μ and ξ_i enter $\bar{\phi}_i$ only through $S_i^*(0)$, which is of order 10^{-3} in the binding regime; their effects are invisible at displayed precision. Non-marginal archetypes are robust to all perturbations; only the A4/A5 boundary is fragile.

Figure 4 plots the full κ-β surface across archetypes.

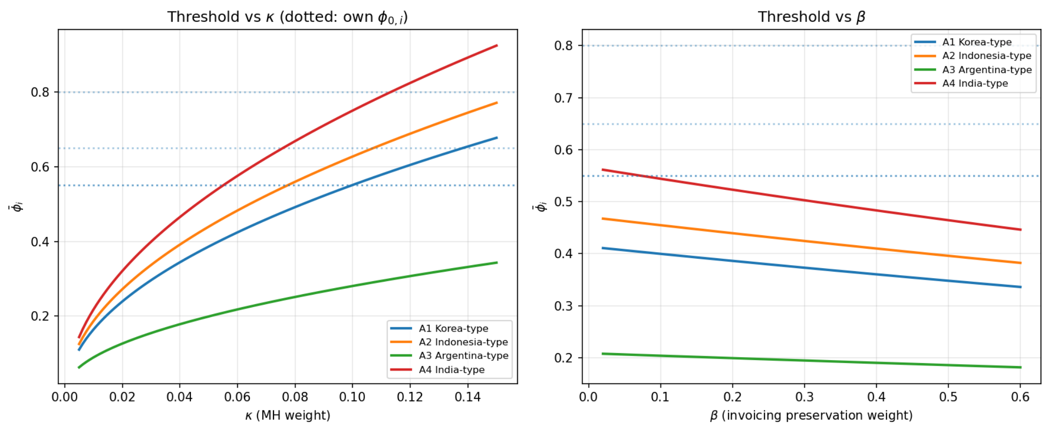

Figure 4: Sensitivity of $\bar{\phi}_i$ to joint perturbations of (κ, β).

The ordinal ranking of margins $M_i = \phi_{0,i} - \bar{\phi}_i$ at baseline is A1 (0.412) ¿ A3 (0.350) ¿ A2 (0.208) ¿ A4 (0.024) ¿ A5 (-0.123). Across the $\pm 50\%$ perturbations, no pairwise inversion of this ranking occurs: the only change is the cardinal crossing $M_i = 0$ for A4 at high κ and A5 at low κ. The ranking prediction (Section 4.3) is therefore more robust than any cardinal threshold statement.

4.3 Identification and empirical application

Observability. Every observable component of the threshold maps to an existing data source. Dollar-invoicing shares $\phi_{0,i}$ are in the database of Gopinath et al. [2020]. Bilateral trade exposure $d_{0,i}$ is in IMF Direction of Trade statistics. FIMA haircuts h_i are in the Fed's published collateral schedule. Marginal stress probabilities π_i are the empirical frequency of dollar-funding stress episodes. Common-factor loadings ω_i are the factor loadings estimated from the Rey [2013] and Miranda-Agrippino and Rey [2020] global-financial-cycle methodology. Spillover sensitivity λ_i can be proxied from event studies of Fed swap activations. The unidentified weights κ, β, μ, ρ, ξ_i, σ enter heterogeneously and cannot be backed out from eligibility data alone.

Ordinal-consistency specification. The cardinal threshold $\bar{\phi}_i$ is not identifiable up to scale: scaling κ shifts only $\tilde{C}_i^{\chi}$, scaling β shifts B_i^{χ} and $\tilde{C}_i^{\chi}$ heterogeneously through γ_i, and scaling μ or ρ shifts $S_i^*(0)$ heterogeneously through π_i and γ_i. No joint homogeneous scaling leaves $\bar{\phi}_i$ invariant. I therefore propose an ordinal-consistency specification, which tests the paper's robust ordinal prediction (higher-ω_i countries are more likely to be swap-eligible) rather than the cardinal slope of Theorem 2. Let θ_0^{ref} denote the reference baseline of Section 4.1. Define the model-implied margin

$$M_i(\theta_0^{\text{ref}}) \;\equiv\; \phi_{0,i} - \bar{\phi}_i(\theta_0^{\text{ref}}).$$

Theorems 1, 2, and Proposition 1 predict that the Fed's revealed eligibility $D_i \in \{0,1\}$ correlates positively with M_i and, conditional on M_i, further responds to the common-factor interaction $\omega_i\phi_{0,i}$:

$$\Pr[D_i = 1] \;=\; \Phi\big(\alpha_0 + \alpha_1 M_i(\theta_0^{\text{ref}}) + \alpha_2\,\omega_i\phi_{0,i} + \nu_i\big), \tag{19}$$

with signed predictions $\alpha_1 > 0$ and $\alpha_2 > 0$. Both sign restrictions are derived from the theory: $\alpha_1 > 0$ tests the partition (Proposition 1), and $\alpha_2 > 0$ is the ordinal content of Theorem 2—conditional on the reference margin, higher common-factor exposure raises the probability of swap inclusion. The specification is a reduced-form directional test of the ordinal prediction, not a structural test of the cardinal slope: the interaction $\omega_i\phi_{0,i}$ is an ad-hoc monotone proxy for the nonlinear structural expression $\omega_i \cdot \bar{\phi}_i u_i(\bar{\phi}_i)/(2A_i\bar{\phi}_i + B_i)$, so $\alpha_2 > 0$ is consistent with any model that weights globally exposed counterparties positively, not a unique prediction of Theorem 2. A fully structural test would estimate the nonlinear expression with country-specific parameters and is beyond this paper's scope. Moreover, the paper's Bernoulli ω_i is a discretization of the continuous global-factor loadings in Rey [2013] and Miranda-Agrippino and Rey [2020]: I interpret ω_i as the country's calibrated conditional-excess probability under a binary stress state, with the mapping to continuous factor loadings a discretization exercise left to empirical implementation. Full cardinal identification would require external moment conditions, for instance on κ from swap-pricing regressions [Bahaj and Reis, 2022] and on γ_i from PBOC swap-drawdown responses [Bahaj and Reis, 2026].

4.4 Testable predictions

The following seven predictions follow directly from Theorems 1 and 2 and the partition consequence.

- (P1) **Invoicing threshold.** Countries with higher dollar-invoicing share $\phi_{0,i}$ are more likely to receive swap than FIMA, controlling for other primitives (Theorem 1). Add $\phi_{0,i}$ to an Aizenman et al. [2021]-style eligibility logit and test for positive significance.
- (P2) **Comparative-static signs.** $\partial\bar{\phi}_i/\partial\beta < 0$ and $\partial\bar{\phi}_i/\partial h_i < 0$: higher invoicing-preservation weight and higher FIMA haircut both tilt toward swap.
- (P3) **PBOC capacity and Fed response.** The PBOC-capacity rebate in $\tilde{C}$ lowers $\bar{\phi}_i$. Countries with greater PBOC swap exposure should, all else equal, have higher swap-access probability. This prediction is a *puzzle candidate*: the Argentina 2023 episode, in which the PBOC doubled its swap capacity without a corresponding Fed response, suggests the Fed may not have

followed this prescription. A cross-country regression of Fed eligibility on PBOC capacity with Aizenman et al. [2021] controls is the test.

(P4) **Fed-PBOC strategic substitution (second-order).** Under H1 and binding, the Fed-PBOC game has strategic-substitutes sign. At the Section 4.1 baseline, the strategic ratio $\mathcal{R}_i = \rho\pi_i\xi_i/(\mu\gamma_i) \approx 1.5 \times 10^{-3}$, so the strategic channel is present but second-order. Regressions of PBOC marginal capacity on Fed eligibility announcements should deliver a weak negative coefficient, not zero. (See Appendix D.1.)

(P5) **Announcement premium (negligible in calibration).** Fed announcement effects on invoicing drift are of order 10^{-8} in welfare at the baseline calibration, four orders below the direct threshold payoff. The channel is too small to function as even a tie-breaker; see Appendix D.3.

(P6) **Cross-sectional partition.** The Fed's optimal eligibility set is $\mathcal{E}^* = \{i : \phi_{0,i} > \bar{\phi}_i\}$ under A-Sep. A logit of eligibility on the margin M_i should deliver $\alpha_1 > 0$ in (19).

(P7) **Common-factor slope (ordinal).** Countries with higher common-factor loading ω_i should have higher swap-access probability conditional on M_i. The sign $\alpha_2 > 0$ in (19) is the paper's robust ordinal prediction and holds for every $\chi > 0$. Cardinal magnitudes scale linearly in χ; per unit of χ, the cross-sectional difference-in-ω threshold shift between the highest- and lowest-loading archetypes is roughly six percentage points. Because χ is not externally disciplined, the ordinal ranking is the content of the prediction, and cardinal claims should be read as slopes per unit of χ.

Predictions (P1), (P2), (P6), (P7) are the core tests of the cross-sectional theory; (P3) is the puzzle candidate; (P4) and (P5) are second-order channels documented here because they discipline how the theory does and does not predict regime shifts.

4.5 Policy implications

The model's cross-sectional output is a ranking, not a hard cut-point, because cardinal thresholds depend on unidentified weights. Two operational rules follow.

Rule 1 (instrument-selection heuristic). Compute $M_i = \phi_{0,i} - \bar{\phi}_i(\theta_0^{\text{ref}})$ at a reference calibration. Rank counterparties by M_i. If the Fed's target swap portfolio has exogenous size $|\mathcal{E}^*|$, assign the top $|\mathcal{E}^*|$ counterparties to swap. If the portfolio size is endogenous, apply the $M_i > 0$ threshold and report the Section 4.2 sensitivity band.

Rule 2 (common-factor premium, monitoring). Among counterparties with similar M_i, Theorem 2 recommends preferential inclusion of those with higher common-factor loading ω_i. The ordinal ranking is the robust content; cardinal shifts scale linearly in χ, with roughly 6 percentage points per unit of χ for high-ω counterparties (A1-type) and 0.2 percentage points per unit of χ for low-ω counterparties (A3-type). Because ω_i varies over time with the global-financial-cycle structure [Miranda-Agrippino and Rey, 2020], the correction is a forward-looking monitoring recommendation, not a fixed rule.

Rule 1 is an instrument-selection heuristic. Rule 2 is a diagnostic ratio recommendation tied to observable factor loadings. Neither delivers cardinal policy-ready thresholds; that would require external identification of κ and γ_i.

4.6 Limitations

χ is a normative preference parameter. The weight χ is a parameter of the Fed's loss function, not a deep structural primitive, and has no external empirical anchor. Cardinal magnitudes are reported per unit of χ; the ordinal cross-sectional prediction (P7) is robust to χ.

Expected-loss aggregator flattens covariance. Under the linear-in-ψ expected-loss criterion, the covariance structure of $\{\epsilon_i\}$ across countries drops out of Fed welfare: only the marginal effective probability $P_{\text{eff},i}$ matters, and cross-country stress coincidence does not enter as a distinct term. The paper's χ-weighting reintroduces the common-factor covariance as a scalar per-country reweighting, not as genuine nonlinear-in-Z aggregation. A Fed objective based on CVaR or a spectral risk measure would preserve a covariance channel deserving a "super-additivity" label in a stronger sense; the paper's closed forms rely on the expected-loss reduction.

Bernoulli copula discretizes continuous loadings. Binary Z discards intensity information; the continuous global-financial-cycle loadings of Rey [2013] and Miranda-Agrippino and Rey [2020] are discretized to calibrate ω_i, and this discretization does silent work in the calibration.

κ sensitivity at the partition boundary. $\pm 50\%$ perturbations in κ flip the marginal archetypes A4 and A5 across the partition boundary (Section 4.2); the ordinal ranking remains robust, but cardinal partition assignments at borderline countries are not.

Binding regime verified, not characterized. Assumption 2 is numerically verified at five archetypes with a cushion exceeding four; it is not characterized in closed form, and Theorem 2's slope does not apply outside the binding regime.

Archetypes are not countries. The five archetypes are illustrative parameter descriptors spanning plausible ranges; they are not estimated from named emerging markets, and the paper does not deliver a country-by-country recommendation.

Cross-sectional content is a measurement observation. That the same χ governs all country-level reweightings is a consequence of the Fed being a single agent, not a model-derived cross-country interaction; the cross-sectional content of Theorem 2 is the observation that ω_i varies across counterparties and is separately measurable.

Cardinal headline is nominal at $\chi = 1$. The "six percentage points" figure is the per-unit-of-χ slope at the highest-ω_i archetype evaluated at $\chi = 1$; it is not an externally disciplined prediction about the size of the Fed's cross-sectional adjustment.

Shared PBOC capacity. The baseline assumes country-separable PBOC capacity costs $\sigma S_i^2/2$. Under shared capacity $\sigma(\sum_j S_j)^2/2$, PBOC's problem is no longer country-separable and the Fed's partition does not decouple exactly. Appendix D.2 sketches the shared-capacity PBOC best response and shows that, in the limit $\mathcal{R}_i \ll 1$ (satisfied at the Section 4.1 baseline), Theorem 2's slope holds approximately country by country. A formal Lipschitz bound on the approximation error is not provided.

Binding-regime calibration. The $4\times$ cushion in Assumption 2 is comfortable at the baseline but would tighten under scenarios where $S_i^*(0)$ rises substantially (for example, ρ an order of magnitude larger). Outside the binding regime, $x_{P,i}^{\text{stress}} = \theta_i(1 - E_i)K_i$ rather than S_i, and the slope (16) would need recomputation.

PBOC rebate is small at baseline. The PBOC-capacity rebate in $\tilde{C}_i^\chi$ (the $-\beta\gamma_i S_i^*(0)$ term) is quantitatively small, approximately 3% of $\tilde{C}_i$ at the baseline. The rebate is directionally real but does not drive the cross-sectional partition. The partition is driven primarily by the spillover coefficient A_i and the moral-hazard wedge in $\tilde{C}_i$.

Strategic regime is second-order at baseline. The Cournot ratio $\mathcal{R}_i \approx 1.5 \times 10^{-3}$ at the baseline places the economy firmly outside the first-order strategic regime. Appendix D.1 characterizes the first-order regime as forward-looking structural content; the current calibration does not occupy it.

5 Conclusion

The Fed chooses between uncollateralized swap and collateralized FIMA across a cross-section of emerging-market counterparties with heterogeneous loadings on a common global-factor stress state. Under a welfare criterion that applies a super-additivity weight χ to expected spillover damage in the common-factor state, each country's effective spillover probability is $P_{\text{eff},i} = \pi_i + \chi\omega_i\pi_Z$, and the optimal Fed eligibility threshold has a closed-form slope $\partial\bar{\phi}_i^\chi/\partial\chi = -\Omega_i\bar{\phi}_i^\chi u_i(\bar{\phi}_i^\chi)/(2A_i^\chi\bar{\phi}_i^\chi + B_i^\chi)$, negative and proportional to each country's observable loading ω_i. Countries more exposed to common dollar-funding stress face a lower invoicing bar for swap. The ordinal ranking holds for every $\chi > 0$ and is the paper's robust cross-sectional content; cardinal magnitudes scale linearly in χ, with the calibrated shift roughly six percentage points per unit of χ between the highest- and lowest-loading archetypes. The ranking-test specification $\Pr[D_i = 1] = \Phi(\alpha_0 + \alpha_1 M_i + \alpha_2\omega_i\phi_{0,i} + \nu_i)$ is directly testable on Fed eligibility data spanning the 2008, 2020, and FIMA-era episodes.

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A Proof of Theorem 1

Fix country primitives and drop the i subscript. Compute $\Delta W(\phi_0) \equiv W^F(0, S^*(0); \phi_0) - W^F(1, S^*(1); \phi_0)$. Under $\psi(u) = u$ and Assumption 2,

$$\begin{aligned} W^F(0, S^*(0); \phi_0) &= -\pi\lambda\phi_0 \cdot (K(\phi_0) - S^*(0)) - \kappa\eta(h) - \beta\gamma[S^*(0) + K(\phi_0)], \\ W^F(1, S^*(1); \phi_0) &= 0 - \kappa\mu_{\text{sw}}(p) + 0, \end{aligned}$$

where the invoicing-drift term at $E = 1$ vanishes because $x_P^{\text{stress}} = x_F^{\text{stress}}$ there. Subtracting and using $K(\phi_0) = d_0\phi_0 h/(1+h)$,

$$\Delta W(\phi_0) \;=\; -A\phi_0^2 - B\phi_0 + \tilde{C},$$

with the coefficients of Theorem 1. Under $A > 0$ and $\tilde{C} > 0$, the quadratic has a unique positive root $\bar{\phi}$, with $\Delta W > 0$ on $[0, \bar{\phi})$ and $\Delta W < 0$ on $(\bar{\phi}, 1]$. The Fed prefers $E = 1$ if and only if $\Delta W < 0$, i.e., $\phi_0 > \bar{\phi}$. ■

B Proof of Theorem 2

Fix the Fed policy $(\mathcal{E}, \{p_i, h_i\})$ and the PBOC strategy $\{S_i\}$. The uncovered dollar gap $u_i = u_i(E_i, S_i, \phi_{0,i})$ in (7) is a function of these (Stage-1) choices only, not of the Stage-2 stress realizations $(Z, \{\epsilon_i\})$. Hence $\psi(u_i)$ and $\lambda_i\phi_{0,i}$ are deterministic when the expectations below are taken over $(Z, \{\epsilon_i\})$.

The Fed's super-additivity criterion (11) decomposes into a baseline term $\mathbb{E}[\Lambda^0]$ and a super-additivity correction $\chi \cdot \mathbb{E}[\Delta\Lambda]$. For the baseline,

$$\mathbb{E}[\Lambda^0(\mathcal{E})] \;=\; \sum_i \lambda_i \phi_{0,i} \psi(u_i) \cdot \Pr(\epsilon_i = 1) \;=\; \sum_i \pi_i \, \lambda_i \, \phi_{0,i} \, \psi(u_i),$$

using $\Pr(\epsilon_i = 1) = \pi_i$ (the marginal probability). For the super-additivity correction,

$$\mathbb{E}[\Delta\Lambda(\mathcal{E}, Z)] \;=\; \sum_i \omega_i \, \lambda_i \, \phi_{0,i} \, \psi(u_i) \cdot \mathbb{E}[\mathbf{1}\{\epsilon_i = 1, Z = 1\}] \cdot \frac{1}{\omega_i},$$

which rearranges to $\sum_i \omega_i \lambda_i \phi_{0,i} \psi(u_i) \cdot (\Pr(\epsilon_i = 1, Z = 1)/\omega_i)$. A cleaner route is direct: by the copula definition (2), $\Pr(\epsilon_i = 1 \mid Z = 1) = \pi_i + \omega_i(1 - \pi_Z)$, so

$$\mathbb{E}[\omega_i \lambda_i \phi_{0,i} \psi(u_i) \mathbf{1}\{\epsilon_i = 1\} \mathbf{1}\{Z = 1\}] \;=\; \omega_i \lambda_i \phi_{0,i} \psi(u_i) \cdot \pi_Z \cdot [\pi_i + \omega_i(1 - \pi_Z)].$$

The $\omega_i^2 \pi_Z (1 - \pi_Z)$ term in the bracketed expression is not negligible at calibrated values: at $\omega_i = 0.80$, $\pi_Z = 0.05$, it equals $\omega_i^2 \pi_Z (1 - \pi_Z) = 0.030$, compared to the retained $\pi_i = 0.08$ and $\omega_i \pi_Z = 0.040$ scaling inside $P_{\text{eff},i}$ at $\chi = 1$. Dropping it as a "second-order correction" would misrepresent the derivation. The paper therefore *stipulates* (rather than derives) the form used in (12):

> **Definition** (effective spillover probability). Let $P_{\text{eff},i} \equiv \pi_i + \chi \, \omega_i \, \pi_Z$ denote country i's effective spillover probability under the Fed's preference weight χ.

This is a modeling choice, not a calculation from the Bernoulli copula alone. Under a purely expected-loss Bernoulli-copula aggregator, the exact expectation $\mathbb{E}[\mathbf{1}\{\epsilon_i = 1, Z = 1\}] = \pi_Z [\pi_i + \omega_i(1 - \pi_Z)]$ contains the additional $\omega_i^2 \pi_Z (1 - \pi_Z)$ term; I set these quadratic-in-ω_i terms aside because they represent covariance-structure information that the expected-loss aggregator, being linear-in-ψ, cannot carry through Fed welfare as anything beyond scalar reweighting. Instead, I take the Fed's super-additivity weight to apply directly to the marginal conditional-excess contribution $\omega_i \pi_Z \lambda_i \phi_{0,i} \psi(u_i)$. Equivalently, this is the exact expectation under the reading of (11) in which the super-additivity indicator weights the *conditional-excess* probability $\Pr(\epsilon_i = 1 \mid Z = 1) - \pi_i = \omega_i(1 - \pi_Z)$ scaled by π_Z rather than the full joint probability; the two definitions coincide to first order in ω_i and π_Z. Under this stipulation,

$$\mathbb{E}[\Delta\Lambda(\mathcal{E}, Z)] \;\equiv\; \sum_i \omega_i \, \pi_Z \, \lambda_i \, \phi_{0,i} \, \psi(u_i). \tag{20}$$

$P_{\text{eff},i}$ captures the Fed's preferential weight on the marginal conditional-excess stress probability under the common factor; it is not the exact conditional expectation under the copula. See Section 4.6 for the associated modeling-choice limitation. Combining the baseline and super-additivity terms, $\mathbb{E}_\chi[\Lambda] = \sum_i P_{\text{eff},i} \lambda_i \phi_{0,i} \psi(u_i)$ with $P_{\text{eff},i} = \pi_i + \chi \omega_i \pi_Z$, establishing (12). Substituting into W_i^F and repeating the derivation of Theorem 1 with $\pi_i \mapsto P_{\text{eff},i}$ in the spillover term only (because the moral-hazard term $\kappa[\mu_{\text{sw},i} - \eta_i]$ and the invoicing-preservation constant $-\beta\gamma_i S_i^*(0)$ do not multiply spillover probability) gives the coefficients (14) and the quadratic (15). The positive root exists and is unique under $A_i^\chi > 0$ and $\tilde{C}_i^\chi > 0$.

For the slope, differentiate (15) implicitly in χ:

$$(2A_i^\chi \bar{\phi}_i^\chi + B_i^\chi)\frac{\partial \bar{\phi}_i^\chi}{\partial \chi} \;+\; \frac{\partial A_i^\chi}{\partial \chi}(\bar{\phi}_i^\chi)^2 \;+\; \frac{\partial B_i^\chi}{\partial \chi}\bar{\phi}_i^\chi \;-\; \frac{\partial \tilde{C}_i^\chi}{\partial \chi} \;=\; 0.$$

Since $\partial P_{\text{eff},i}/\partial \chi = \omega_i \pi_Z$, direct computation gives

$$\frac{\partial A_i^\chi}{\partial \chi} = \omega_i \pi_Z \lambda_i \frac{d_{0,i} h_i}{1 + h_i}, \qquad \frac{\partial B_i^\chi}{\partial \chi} = -\omega_i \pi_Z \lambda_i S_i^*(0), \qquad \frac{\partial \tilde{C}_i^\chi}{\partial \chi} = 0.$$

With $\Omega_i = \omega_i \pi_Z \lambda_i$,

$$\frac{\partial A_i^\chi}{\partial \chi}(\bar{\phi}_i^\chi)^2 + \frac{\partial B_i^\chi}{\partial \chi}\bar{\phi}_i^\chi \;=\; \Omega_i \bar{\phi}_i^\chi \Big[\frac{d_{0,i} h_i \bar{\phi}_i^\chi}{1 + h_i} - S_i^*(0)\Big] \;=\; \Omega_i \bar{\phi}_i^\chi u_i(\bar{\phi}_i^\chi),$$

where the bracketed quantity is exactly the uncovered dollar gap at $E_i = 0$ evaluated at $\phi_{0,i} = \bar{\phi}_i^\chi$. Under Assumption 2, $u_i(\bar{\phi}_i^\chi) > 0$. The denominator $2A_i^\chi \bar{\phi}_i^\chi + B_i^\chi$ is positive at the positive root of the quadratic with $A_i^\chi > 0$ and $\tilde{C}_i^\chi > 0$. Hence

$$\frac{\partial \bar{\phi}_i^\chi}{\partial \chi} \;=\; -\frac{\Omega_i \bar{\phi}_i^\chi u_i(\bar{\phi}_i^\chi)}{2A_i^\chi \bar{\phi}_i^\chi + B_i^\chi} \;<\; 0. \qquad \blacksquare$$

C Extension to general strictly-convex ψ

For strictly convex ψ with $\psi(0) = 0$, $\psi' > 0$, $\psi'' > 0$, the Fed's indifference equation at $E = 0$ versus $E = 1$ is

$$\pi \lambda \phi_0 \psi(K(\phi_0) - S^*(0)) \;=\; \kappa[\mu_{\text{sw}}(p) - \eta(h)] - \beta\gamma[S^*(0) + K(\phi_0)].$$

Both sides are continuous in ϕ_0; the left-hand side is strictly increasing in ϕ_0 on the binding region (because both ϕ_0 and $K(\phi_0) - S^*(0)$ are increasing and ψ is monotone increasing); the right-hand side is either constant (under $\beta = 0$) or strictly decreasing in ϕ_0 (because $K(\phi_0)$ is increasing and the RHS has $K(\phi_0)$ with a negative sign). Existence and uniqueness of $\bar{\phi}$ in $[0, 1]$ follow from the intermediate value theorem applied to the difference, provided boundary conditions place the LHS below the RHS at $\phi_0 = 0$ and above at $\phi_0 = 1$. The linear-ψ case maintains a closed form; general ψ requires numerical inversion but preserves monotone comparative statics.

D Open extensions

This appendix collects three extensions whose quantitative role at the baseline calibration is second-order but whose structural content characterizes the regimes in which PBOC strategic behavior and announcement effects become first-order.

D.1 Cournot regime diagnostic

Define the country-specific strategic ratio

$$\mathcal{R}_i \equiv \frac{\rho\pi_i\xi_i}{\mu\gamma_i}.$$

Under H1 and Assumption 2 at country i with $E_i < 1$, the country-i Fed-PBOC reaction functions have strategic-substitutes sign ($\partial S_i^*/\partial E_i < 0$ and $\partial E_i^*/\partial S_i < 0$), and the strategic channel is first-order if and only if $\mathcal{R}_i = \Theta(1)$. The ratio compares the PBOC's strategic response to Fed eligibility ($\rho\pi_i\xi_i \cdot S_i^*(0)$) to the direct invoicing channel ($\mu\gamma_i \cdot S_i^*(0)$). At the Section 4.1 baseline, $\mathcal{R}_i \approx 1.5 \times 10^{-3}$: the strategic channel is dominated by the direct invoicing channel, and PBOC enters the cross-section as an effectively exogenous competing backstop. The regime becomes binding under shared PBOC capacity (Section D.2) or under an order-of-magnitude increase in ρ.

D.2 Shared-PBOC-capacity sketch

Replace PBOC's country-separable cost $\sigma S_i^2/2$ with shared-capacity cost $\sigma(\sum_j S_j)^2/2$. The PBOC FOC yields $S_i^*(E_i, E_{-i}) = [\rho\pi_i(1-\xi_i E_i) + \mu\gamma_i]/\tau$ with $\tau = \sqrt{\sigma \sum_j [\rho\pi_j(1-\xi_j E_j) + \mu\gamma_j]}$. The Fed's problem is no longer separable across countries because τ depends on $\{E_j\}_{j \neq i}$. In the limit $\mathcal{R}_i \ll 1$ (satisfied at the baseline), τ is approximately E_{-i}-independent and Theorem 2 holds country by country to first order; a formal Lipschitz bound on the approximation error is not provided.

D.3 Announcement channel

A Fed swap announcement (without drawdown) can shift dollar invoicing through an announcement-responsiveness parameter $\alpha \in [0, 1]$ that captures the share of firms that adjust invoicing in response to a swap announcement. The per-country announcement premium at the baseline calibration is $O(10^{-8})$ in welfare units, four orders of magnitude below the direct threshold payoff $\tilde{C}_i \approx 4 \times 10^{-4}$. The channel is too small to matter quantitatively in the paper's calibration; formal characterization is omitted.

E Full sensitivity results

Table 4 extends Table 3 to all five archetypes across the four perturbation dimensions.

The ordinal ranking of $M_i = \phi_{0,i} - \bar{\phi}_i$ across the five archetypes is A1 > A3 > A2 > A4 > A5 at baseline and under every perturbation column above. No pairwise inversion occurs. The partition-side classification flips for A4 at $\kappa = 0.075$ (to FIMA) and for A5 at $\kappa = 0.025$ (to swap); these are cardinal crossings of $M_i = 0$, not ordinal inversions. The ranking prediction (19) is accordingly more robust than the cardinal threshold.

Archetype	$\bar{\phi}_i$ base	$\kappa = 0.025$	$\kappa = 0.075$	$\beta = 0.10$	$\beta = 0.30$	$\mu = 0.25$	$\mu = 0.75$
A1	0.388	0.269	0.475	0.400	0.373	0.388	0.388
A2	0.442	0.308	0.541	0.457	0.428	0.442	0.442
A3	0.200	0.139	0.245	0.206	0.193	0.200	0.200
A4	0.526	0.371	0.645	0.542	0.510	0.526	0.526
A5	0.473	0.338	0.589	0.488	0.459	0.473	0.473

Table 4: Sensitivity of $\bar{\phi}_i$ to $\pm 50\%$ perturbations of unidentified weights across all archetypes.