Common Benchmark Load and Price Fragility

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Abstract

When does benchmark customization stabilize markets, and when does it amplify crowding? This paper studies a delegated asset-management economy in which asset owners choose benchmark intensity, managers trade with tracking-error concerns, and benchmark-sensitive flows amplify demand for a common characteristic. The model yields a sufficient statistic for fragility: average common benchmark overlap. On the positive-price branch, price loading is finite if and only if overlap stays below a sharp threshold, and the loading becomes convex as overlap rises. A planner chooses less overlap than private investors because private benchmark design ignores how overlap steepens price impact. Benchmark heterogeneity stabilizes prices only when it lowers common load on the crowded characteristic. The paper is a characterization result. It does not claim a new benchmarking mechanism. It isolates the state variable that governs fragility inside a familiar delegated-demand environment.

1 Introduction

Delegated capital now sits behind a large share of asset demand, and much of that capital is benchmarked. This institutional fact raises a narrow but important question: when does benchmark customization disperse risk-bearing, and when does it instead concentrate demand on the same traded object? Existing theory already shows that benchmarking distorts delegated demand, that benchmarking can arise endogenously from contracting, and that heterogeneous benchmarks can generate comovement and price pressure. What remains less well characterized is the state variable that governs fragility once many investors load their benchmarks on the same characteristic.

This paper studies a parsimonious delegated asset-pricing model with one benchmarkable characteristic. Asset owners choose benchmark intensity, managers trade with tracking-error concerns, and benchmark-sensitive flows amplify positions that load on the common benchmarked characteristic. The model delivers a sharp characterization result: average common benchmark overlap is the sufficient statistic for fragility on the positive-price branch. Price loading rises with overlap, rises convexly in overlap, and becomes arbitrarily large as overlap approaches an explicit threshold. A planner chooses less overlap than private investors because each owner treats the benchmark-design problem privately and ignores how common overlap steepens equilibrium price impact. The main text keeps benchmark choice reduced form to isolate that object cleanly. Appendix A.1 then shows that once owners value delegated exposure directly, the private benchmark choice depends on the same fragility forces that drive the trading equilibrium.

The paper does not claim that benchmarking affects prices for the first time. That claim is already in the literature. Leippold and Rohner (2011) and Cuoco and Kaniel (2011) show that benchmark-linked delegated contracts distort demand and equilibrium prices. Buffa, Vayanos, and Woolley (2022) show that heterogeneous benchmarks generate comovement in equilibrium, and Buffa, Vayanos, and Woolley (2023) show that benchmarking intensity is the key sufficient statistic for price effects. Kashyap, Kovrijnykh, Li, and Pavlova (2023) show that benchmarking can arise endogenously and that private investors choose too much of it. The contribution here is narrower. The model compresses these forces into a tractable one-characteristic environment and derives an exact condition for when common overlap becomes a fragility state variable. The paper also shows that benchmark heterogeneity stabilizes prices if and only if it lowers common loading on the crowded characteristic. That exact characterization matters because it identifies the statistic that richer delegated-benchmarking models must preserve if they are to generate overlap-driven fragility.

That narrow contribution comes with two deliberate choices. First, the benchmark-design stage in the main text is reduced form. Owners choose benchmark intensity from a simple private objective rather than from a full delegated-contracting problem. This choice gives up microfoundation depth in exchange for a cleaner characterization of overlap and fragility. Appendix A.1 shows that the same fragility forces enter the private benchmark-choice condition once owners value delegated exposure directly. Second, the heterogeneity result is secondary. The model does not solve a fully general multi-characteristic benchmarking equilibrium. Appendix A.2 states a minimal richer environment in which common load remains sufficient and identifies the separability conditions behind that result.

The computational exploration disciplines the sales pitch. The main theorem holds exactly, and the nonlinear region is economically large when overlap moves close to the threshold. At the same time, the baseline illustrative calibration sits far from the singularity. The paper therefore does not argue that benchmark fragility is always severe. It argues that fragility becomes severe in high-overlap states, and it identifies the object that determines when those states arise. In the model, those states correspond to stronger benchmark-sensitive flows, lower outside depth, and greater harmonization of benchmark design across investors.

The paper proceeds as follows. Section 2 states the environment, the benchmark-design problem, and the positive-price branch that matches the crowded-characteristic application. Section 3 derives the price-loading formula, the overlap threshold, a local-stability result that selects the positive-price branch, the planner wedge, and the tracking-intensity sign condition. Section 4 interprets the results, relates them to the existing benchmarking literature, and states the model's limitations. The appendix shows how a richer owner objective makes private benchmark choice depend on equilibrium fragility forces, states a minimal richer environment in which common load remains sufficient, and reports the calibration and overlap figure used in the computational exploration.

Section 5 concludes.

2 Model

There is a unit mass of asset owners indexed by $j \in [0, 1]$. Each owner delegates portfolio choice to one manager. The only traded object is exposure to a benchmarkable characteristic portfolio x, normalized so that one unit of portfolio exposure equals one unit of loading on that characteristic.

The timeline has three dates.

1. Each owner chooses a benchmark coefficient $b_j \geq 0$.
2. Managers observe the benchmark profile and choose characteristic exposure z_j.
3. Markets clear at price loading β, the price per unit of characteristic exposure.

The residual sector supplies the characteristic portfolio with linear inverse supply

$$Z^S(\beta) = \lambda\beta, \qquad \lambda > 0,$$

so larger λ means deeper outside risk-bearing capacity. Average benchmark overlap on the common characteristic is

$$\bar{b} \equiv \int_0^1 b_j \, dj.$$

Manager j chooses z_j to maximize

$$U_j(z_j; \beta, b_j, \bar{b}) = (m - \beta)z_j - \frac{\gamma}{2}z_j^2 - \frac{\tau}{2}(z_j - b_j)^2 + \kappa\bar{b}\beta z_j, \tag{1}$$

where $m > 0$ is the characteristic payoff loading, $\gamma > 0$ is portfolio curvature, $\tau > 0$ is tracking intensity, and $\kappa \geq 0$ measures benchmark-sensitive flow amplification. The term $\kappa\bar{b}\beta z_j$ captures the idea that a manager with more benchmark exposure benefits more from flow-sensitive continuation value when common overlap is high.

Each owner chooses b_j before trading. The private benchmark-design problem is

$$\max_{b_j \geq 0} \; vb_j - \frac{1}{2}b_j^2, \tag{2}$$

where $v > 0$ summarizes the private value of stronger benchmarking. The owner problem is reduced form. The paper uses it to isolate the equilibrium consequences of overlap rather than to solve a full contracting problem.

With symmetric benchmark design, $b_j = b$, a planner solves

$$\max_{b \geq 0} \; W(b) \equiv vb - \frac{1}{2}b^2 - \frac{c}{2}\beta(b)^2, \qquad c > 0, \tag{3}$$

where the last term captures the social cost of non-fundamental price loading.

The application of interest is the positive-price branch. Because $m > 0$, a positive β corresponds to a crowded long characteristic. Define

$$A \equiv (\gamma + \tau)\lambda + 1.$$

The analysis below assumes

$$v < \frac{A}{\kappa} \tag{4}$$

when $\kappa > 0$. This restriction keeps the decentralized symmetric outcome on the positive-price branch. If $\kappa = 0$, the threshold is infinite and the restriction is vacuous.

An equilibrium is a benchmark profile $\{b_j\}_{j\in[0,1]}$, a manager allocation profile $\{z_j\}_{j\in[0,1]}$, and a price loading β such that: (i) each manager solves (1); (ii) each owner solves (2); and (iii) aggregate demand equals residual supply.

3 Results

Proposition 1 (Manager demand and price loading). *Fix a benchmark profile $\{b_j\}_{j\in[0,1]}$ with average overlap $\bar{b}$. Manager j chooses*

$$z_j^*(b_j, \bar{b}) = \frac{m + \tau b_j - (1 - \kappa\bar{b})\beta}{\gamma + \tau}.$$

Aggregate demand is

$$\bar{z} = \frac{m + \tau\bar{b} - (1 - \kappa\bar{b})\beta}{\gamma + \tau}.$$

Market clearing implies

$$\beta(\bar{b}) = \frac{m + \tau\bar{b}}{A - \kappa\bar{b}} \tag{5}$$

whenever $A - \kappa\bar{b} \neq 0$. Because $m + \tau\bar{b} > 0$, the positive-price branch exists if and only if $A - \kappa\bar{b} > 0$.

Proof. Differentiate (1) with respect to z_j:

$$\frac{\partial U_j}{\partial z_j} = m - \beta - \gamma z_j - \tau(z_j - b_j) + \kappa\bar{b}\beta.$$

Set the derivative equal to zero and collect terms:

$$(\gamma + \tau)z_j = m + \tau b_j - (1 - \kappa\bar{b})\beta.$$

This gives the stated manager demand. Integrating over owners yields

$$\bar{z} = \int_0^1 z_j^* dj = \frac{m + \tau\bar{b} - (1 - \kappa\bar{b})\beta}{\gamma + \tau}.$$

Impose market clearing, $\bar{z} = \lambda\beta$:

$$\lambda\beta = \frac{m + \tau\bar{b} - (1 - \kappa\bar{b})\beta}{\gamma + \tau}.$$

Multiply by $\gamma + \tau$ and rearrange:

$$\beta\left((\gamma + \tau)\lambda + 1 - \kappa\bar{b}\right) = m + \tau\bar{b}.$$

Using $A = (\gamma + \tau)\lambda + 1$ gives (5). Because the numerator is strictly positive, the sign of β matches the sign of $A - \kappa\bar{b}$. The positive-price branch therefore exists exactly when $A - \kappa\bar{b} > 0$. □

Equation (5) is the paper's central object. Overlap does not just shift demand. It changes the denominator of equilibrium price impact.

Proposition 2 (Positive-price fragility threshold)**.** *On the positive-price branch,*

$$0 \le \bar{b} < \frac{A}{\kappa}$$

for $\kappa > 0$, price loading satisfies

$$\beta'(\bar{b}) = \frac{\tau A + \kappa m}{(A - \kappa\bar{b})^2} > 0$$

and

$$\beta''(\bar{b}) = \frac{2\kappa(\tau A + \kappa m)}{(A - \kappa\bar{b})^3} > 0.$$

Price loading is increasing and convex in overlap. As $\bar{b} \uparrow A/\kappa$, $\beta(\bar{b}) \to +\infty$.

Proof. Differentiate (5) with the quotient rule:

$$\beta'(\bar{b}) = \frac{\tau(A - \kappa\bar{b}) - (m + \tau\bar{b})(-\kappa)}{(A - \kappa\bar{b})^2} = \frac{\tau A + \kappa m}{(A - \kappa\bar{b})^2}.$$

Differentiate again:

$$\beta''(\bar{b}) = (\tau A + \kappa m)\frac{d}{d\bar{b}}(A - \kappa\bar{b})^{-2} = \frac{2\kappa(\tau A + \kappa m)}{(A - \kappa\bar{b})^3}.$$

Both derivatives are positive on the positive-price branch. The numerator of (5) stays positive as $\bar{b} \uparrow A/\kappa$, while the denominator converges to zero from above, so $\beta(\bar{b})$ diverges. □

The proposition gives a threshold for fragility on the positive-price branch, not a global equilibrium-existence result. The negative-price algebraic branch remains outside the intended crowded-characteristic application.

Proposition 3 (Local stability selects the positive-price branch)**.** *Fix a benchmark profile with average overlap $\bar{b}$ and consider the price-adjustment process*

$$\dot{\beta} = \xi\left(\bar{z}(\beta) - \lambda\beta\right), \qquad \xi > 0,$$

where $\bar{z}(\beta)$ is aggregate demand from Proposition 1. The unique algebraic equilibrium is locally stable if and only if

$$A - \kappa\bar{b} > 0.$$

Proof. Proposition 1 implies

$$\bar{z}(\beta) - \lambda\beta = \frac{m + \tau\bar{b} - (1 - \kappa\bar{b})\beta}{\gamma + \tau} - \lambda\beta = \frac{m + \tau\bar{b}}{\gamma + \tau} - \frac{A - \kappa\bar{b}}{\gamma + \tau}\beta.$$

This excess-demand function is affine in β, so its derivative is

$$\frac{d}{d\beta}\left(\bar{z}(\beta) - \lambda\beta\right) = -\frac{A - \kappa\bar{b}}{\gamma + \tau}.$$

Under the adjustment process above, local stability requires this derivative to be negative. This adjustment rule is the reduced-form counterpart of market makers or price setters moving quotes in the direction of excess demand. Because $\gamma + \tau > 0$, that condition is equivalent to

$$A - \kappa\bar{b} > 0.$$

If $A - \kappa\bar{b} < 0$, the derivative is positive and the algebraic equilibrium is locally unstable under this adjustment process. The positive-price branch is therefore the locally stable branch in the crowded-characteristic application. Any monotone price-adjustment process with the same local excess-demand slope yields the same sign condition. □

Proposition 4 (Private benchmark design is socially excessive)**.** *The private optimum is*

$$b^P = v.$$

Any interior planner optimum on the positive-price branch satisfies

$$v - b^S - c\beta(b^S)\beta'(b^S) = 0.$$

Therefore $b^S < b^P$.

Proof. The private problem (2) has first-order condition

$$v - b_j = 0,$$

so $b^P = v$. The second derivative is -1, so the optimum is unique.

For the planner,

$$W'(b) = v - b - c\beta(b)\beta'(b).$$

Any interior optimum satisfies

$$v - b^S - c\beta(b^S)\beta'(b^S) = 0.$$

On the positive-price branch, Proposition 2 implies $\beta(b^S) > 0$ and $\beta'(b^S) > 0$. Hence

$$b^S = v - c\beta(b^S)\beta'(b^S) < v = b^P.$$

□

The welfare result is a corollary of the price-loading theorem. Private investors ignore the slope effect that their benchmark choice creates for everyone else. Appendix A.1 shows that the same fragility forces enter the private benchmark-design first-order condition once owners value delegated exposure directly.

Corollary 1 (When heterogeneity reduces fragility)**.** *Suppose each owner has fixed benchmark intensity $B > 0$ and chooses how much of it to load on the common crowded characteristic. Let $\theta_j \in [0, 1]$ be the share loaded on the common characteristic, so $b_j = B\theta_j$. Then fragility depends only on*

$$\omega = \int_0^1 b_j dj = B \int_0^1 \theta_j dj.$$

Benchmark heterogeneity lowers fragility if and only if it lowers ω.

Proof. Substitute $b_j = B\theta_j$ into Proposition 1. The price-loading formula becomes

$$\beta(\omega) = \frac{m + \tau\omega}{A - \kappa\omega}.$$

Proposition 2 implies

$$\frac{d\beta}{d\omega} = \frac{\tau A + \kappa m}{(A - \kappa\omega)^2} > 0$$

on the positive-price branch. Any change in benchmark heterogeneity affects fragility only through its effect on ω. □

This corollary identifies the sufficient statistic for fragility. Benchmark labels, mandate variety, and other forms of dispersion do not matter unless they change common load on the crowded object. Appendix A.2 shows that the same sufficient-statistic logic survives in a minimal richer environment with one common characteristic and orthogonal idiosyncratic benchmark dimensions.

Proposition 5 (Tracking intensity has a signed condition)**.** *Holding overlap fixed on the positive-price branch,*

$$\frac{\partial \beta}{\partial \tau} = \frac{\bar{b}(\gamma\lambda + 1 - \kappa\bar{b}) - \lambda m}{(A - \kappa\bar{b})^2}.$$

Tracking intensity raises price loading if and only if

$$\bar{b}(\gamma\lambda + 1 - \kappa\bar{b}) > \lambda m.$$

Proof. Differentiate (5) with respect to τ. Because $\partial A/\partial\tau = \lambda$,

$$\frac{\partial\beta}{\partial\tau} = \frac{\bar{b}(A-\kappa\bar{b}) - \lambda(m+\tau\bar{b})}{(A-\kappa\bar{b})^2}.$$

Substitute $A = (\gamma+\tau)\lambda + 1$ and cancel the $\lambda\tau\bar{b}$ terms:

$$\frac{\partial\beta}{\partial\tau} = \frac{\bar{b}(\gamma\lambda + 1 - \kappa\bar{b}) - \lambda m}{(A-\kappa\bar{b})^2}.$$

The denominator is strictly positive on the positive-price branch, so the sign of the derivative matches the sign of the numerator. □

The exact sign condition matters for interpretation. Tighter benchmarking does not always increase fragility. It does so only when overlap is already large enough relative to fundamentals and outside depth.

4 Discussion

The model yields four testable predictions. First, benchmark revisions should move prices more in securities with greater common benchmark overlap, not simply greater benchmark ownership. Second, the same benchmark revision should have a larger price effect when outside risk-bearing capacity is low. Third, stronger benchmark-sensitive flows should steepen the relation between overlap and price impact. Fourth, mandate customization should stabilize prices only when it lowers common load on the crowded characteristic. Cosmetic customization should not matter.

The computational exploration supports the theory but narrows its scope. Appendix A.4 reports the baseline calibration and Figure 1. With the illustrative calibration, the baseline price loading is modest and the threshold is distant. The model becomes quantitatively dramatic only in high-overlap states. This feature is not a flaw in the theorem, but it matters for interpretation. The paper characterizes when fragility becomes severe. It does not claim that benchmarked delegated capital produces severe fragility in every state.

The paper also gives a concrete interpretation of those high-overlap states. In the model, the economy moves toward the threshold when benchmark-sensitive flows strengthen, when outside risk-bearing capacity falls, or when benchmark design becomes more harmonized across investors. In market terms, the risky region corresponds to a larger passive and benchmark-aware investor base, more common mandates tied to the same benchmark family, and shallower outside balance sheets.

This framing helps locate the paper inside the existing literature. Endogenous benchmarking and planner wedges are already present in Kashyap, Kovrijnykh, Li, and Pavlova (2023). Overlapping benchmarks and equilibrium comovement are already present in Buffa, Vayanos, and Woolley (2022). Benchmarking intensity as a sufficient statistic is already present in Buffa, Vayanos, and

Woolley (2023). The model here contributes a smaller object: a transparent threshold characterization for the positive-price branch and an exact condition for when customization stabilizes rather than destabilizes. That contribution is incremental in mechanism and stronger in characterization. The value of the characterization is that it turns a broad verbal claim about crowding into an exact mapping from benchmark design to fragility. Appendix A.2 makes this point in a minimal extension with orthogonal benchmark dimensions, and Appendix A.3 shows exactly how the mapping fails once cross-flow terms re-enter the problem.

The model also has clear limits. The benchmark-design stage in the main text is reduced form. Owners choose benchmark intensity from a simple private objective rather than from a richer delegated-contracting problem. This choice keeps the overlap theorem transparent. Appendix A.1 shows that once owners value delegated exposure directly, the private benchmark choice responds to the same equilibrium forces that drive fragility. The heterogeneity result is still thin. The paper shows that only common load matters for fragility, but it does not solve a full multi-characteristic equilibrium with rich benchmark dispersion. Appendix A.2 clarifies the separability conditions under which the sufficient-statistic logic survives. A more detailed environment might preserve those conditions or might reveal additional interactions that this scalar setup suppresses.

The reduced-form structure is still useful. It isolates a sufficient statistic that is easy to carry to richer environments and to empirical work. It also disciplines rhetoric. If customization fails to lower common load, the model says that customization should not be expected to stabilize markets. If overlap rises while benchmark-sensitive flows remain strong, the model says that the price system can move from calm to fragile nonlinearly rather than smoothly. Proposition 3 and Appendix A.1 also separate two claims that are often run together: private benchmark design can depend on equilibrium fragility forces, and it can still remain socially excessive because owners do not internalize how overlap steepens price impact for everyone else.

5 Conclusion

This paper studies a delegated asset-management economy in which asset owners choose benchmark intensity and managers trade with tracking concerns and benchmark-sensitive flows. Average common benchmark overlap is the sufficient statistic for fragility on the positive-price branch. Price loading rises convexly with overlap, diverges at a sharp threshold, and gives a planner a reason to choose less overlap than private investors. Benchmark heterogeneity stabilizes prices only when it lowers common load on the crowded characteristic. The paper's contribution is a tractable characterization of these objects inside a familiar benchmarking environment.

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A Appendix

A.1 A microfoundation for private benchmark choice

The main text uses the reduced-form owner problem in equation (2) to isolate the overlap theorem. This appendix shows that the private benchmark choice can depend on the same equilibrium forces that drive fragility.

Suppose symmetric owners choose a common benchmark intensity b and value delegated exposure because higher exposure to the benchmarkable characteristic attracts fee revenue or client capital. Let $\chi > 0$ measure the private payoff from one unit of delegated exposure. The owner's problem is

$$\max_{b\geq 0} \ \chi z^*(b,b) - \frac{1}{2}b^2. \tag{6}$$

On the symmetric equilibrium path, market clearing implies $z^*(b,b) = \lambda\beta(b)$, so the owner solves

$$\max_{b\geq 0} \ \chi\lambda\beta(b) - \frac{1}{2}b^2.$$

Proposition 6 (Private benchmark choice responds to fragility forces)**.** *On the positive-price branch, any interior private optimum for problem* (6) *satisfies*

$$b = \chi\lambda\frac{\tau A + \kappa m}{(A-\kappa b)^2}. \tag{7}$$

The private first-order condition depends directly on flow sensitivity κ, outside depth λ, portfolio curvature γ, and tracking intensity τ through $A = (\gamma+\tau)\lambda + 1$.

Proof. Under symmetry, Proposition 1 gives

$$\beta(b) = \frac{m+\tau b}{A-\kappa b}.$$

Substitute this expression into the owner objective:

$$\chi\lambda\beta(b) - \frac{1}{2}b^2 = \chi\lambda\frac{m+\tau b}{A-\kappa b} - \frac{1}{2}b^2.$$

Differentiate with respect to b:

$$\chi\lambda\beta'(b) - b = 0.$$

Proposition 2 gives

$$\beta'(b) = \frac{\tau A + \kappa m}{(A-\kappa b)^2}.$$

Substitute this derivative into the first-order condition to obtain (7). The dependence on κ, λ, γ, and τ follows from the definition of A and the derivative formula. □

Equation (7) answers the main endogenous-design objection to the reduced-form benchmark-choice stage. Once owners value delegated exposure directly, the private benchmark choice loads more heavily on the crowded characteristic when flow sensitivity is stronger and when the price-loading derivative is steeper. The planner wedge remains because private owners still do not internalize the social cost term in equation (3).

Interiority. The appendix proposition characterizes any interior optimum. Existence and uniqueness depend on the curvature of the private objective. The paper does not need those global conditions for the main overlap theorem. It uses the proposition only to show that once owners value delegated exposure directly, the private benchmark-choice condition depends on the same fragility forces that shape the trading equilibrium.

A.2 A minimal richer environment with a sufficient statistic

The one-characteristic model makes common load sufficient by construction. This subsection states the smallest richer environment in which that sufficiency survives.

Suppose each owner can load a benchmark on one common crowded characteristic x and one orthogonal idiosyncratic characteristic y_j. Let benchmark load on the common characteristic be b_j and let the idiosyncratic benchmark load be h_j. Suppose manager j's objective is additively separable:

$$U_j = U_j^x(z_{xj};\beta_x,b_j,\omega) + U_j^y(z_{yj};\beta_{yj},h_j), \qquad \omega \equiv \int_0^1 b_j\,dj,$$

where the common block U_j^x has the same form as equation (1) and the idiosyncratic block contains no cross-price or cross-flow term with x. Residual supply is also separable across the two blocks.

Proposition 7 (Sufficiency in a separable richer environment)**.** *In the separable two-block environment above, equilibrium loading on the common characteristic is*

$$\beta_x(\omega) = \frac{m_x+\tau\omega}{A_x-\kappa\omega}, \qquad A_x \equiv (\gamma+\tau)\lambda_x + 1,$$

and depends on the benchmark profile only through the average common load ω. Idiosyncratic benchmark loads $\{h_j\}$ affect only the y-block prices.

Proof. Additive separability implies the manager's problem splits into an x-block and a y-block. The first-order condition for the common block is therefore identical to the first-order condition in Proposition 1 after replacing $\bar{b}$ by ω and allowing the common block to have its own payoff and residual-depth parameters, m_x and λ_x. Aggregation over owners then gives

$$\bar{z}_x = \frac{m_x + \tau\omega - (1 - \kappa\omega)\beta_x}{\gamma + \tau}.$$

Separable residual supply implies market clearing in the common block is

$$\bar{z}_x = \lambda_x \beta_x.$$

Solving yields

$$\beta_x(\omega) = \frac{m_x + \tau\omega}{A_x - \kappa\omega}, \qquad A_x = (\gamma + \tau)\lambda_x + 1.$$

Because the y-block is orthogonal and separable, its benchmark loads $\{h_j\}$ do not enter the common-block market-clearing equation. They therefore affect only idiosyncratic prices. □

This proposition clarifies what the scalar model is really using. Common load remains sufficient when benchmark design enters the common crowded characteristic through an additive block and the idiosyncratic benchmark dimensions do not feed back into common-block flows or prices. The sufficient statistic fails once cross-impact, multiple common flow-sensitive characteristics, or non-separable supply re-enters the problem.

A.3 A counterexample when sufficiency fails

The separable theorem above is useful only if the reader can also see where it breaks. This subsection gives the smallest counterexample.

Suppose the common block of manager j's objective becomes

$$U_j^x(z_{xj}; \beta_x, b_j, \omega, h_j, \beta_y) = (m_x - \beta_x)z_{xj} - \frac{\gamma}{2}z_{xj}^2 - \frac{\tau}{2}(z_{xj} - b_j)^2 + \kappa\omega\beta_x z_{xj} + \eta h_j \beta_y z_{xj},$$

where h_j is the owner's idiosyncratic benchmark load and $\eta \neq 0$ measures a cross-flow or cross-price spillover from the idiosyncratic block to the common block.

Proposition 8 (Common load is no longer sufficient with cross-flow spillovers)**.** *If $\eta \neq 0$, aggregate demand for the common characteristic depends on both average common load ω and the cross term*

$$\int_0^1 h_j \beta_y \, dj.$$

Therefore the common price loading cannot, in general, be written as a function of ω alone.

Proof. The first-order condition for the common block is

$$(\gamma + \tau) z_{xj} = m_x + \tau b_j - (1 - \kappa\omega)\beta_x + \eta h_j \beta_y.$$

Integrating over owners gives

$$\bar{z}_x = \frac{m_x + \tau\omega - (1 - \kappa\omega)\beta_x + \eta \int_0^1 h_j \beta_y \, dj}{\gamma + \tau}.$$

The aggregate-demand equation now contains the additional term

$$\eta \int_0^1 h_j \beta_y \, dj,$$

which does not collapse to a function of ω alone unless $\eta = 0$ or the cross term is constant by assumption. Market clearing in the common block therefore depends on more than average common load. □

This counterexample gives the exact scope of the paper's sufficient-statistic claim. Common load is sufficient in separable environments with orthogonal benchmark dimensions. Once cross-flow or cross-price terms connect those dimensions, additional state variables matter.

A.4 Calibration and overlap figure

Table 1 reports the illustrative calibration used in the computational exploration.

Parameter	Value	Interpretation
m	0.04	Characteristic payoff loading
γ	2.0	Portfolio curvature
τ	0.6	Tracking intensity
λ	2.0	Residual risk-bearing depth
κ	1.2	Flow sensitivity
$\bar{b}$	0.8	Common benchmark overlap

Table 1: Illustrative calibration for the exploration exercise.

At this calibration, the baseline price loading is $\beta = 0.099$, the positive-price threshold is $\bar{b}^{\star} = 5.167$, and the economy sits 4.367 overlap units below the singularity. The nonlinear region is therefore economically important in high-overlap states rather than at the baseline. In the model, those high-overlap states correspond to stronger benchmark-sensitive flows, lower outside depth, or more harmonized benchmark design across investors.

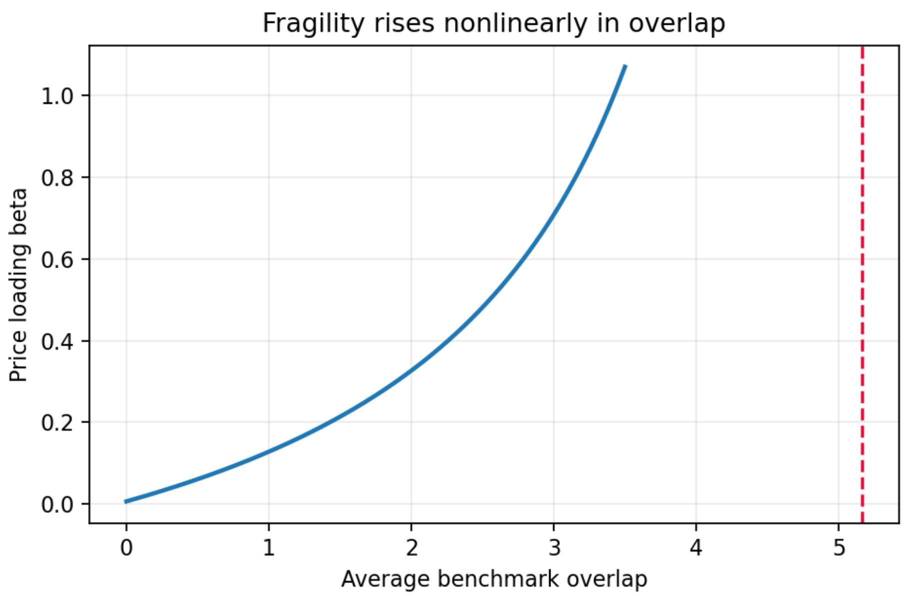

Figure 1: Price loading rises nonlinearly with common benchmark overlap and becomes steep near the threshold.